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CEPHEIDS AND LONG-PERIOD VARIABLES IN NGC 4395

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ABSTRACT

Repeated imaging observations of NGC 4395 were made with the WIYN 3.5 m and the KPNO 2.1 m telescopes. From the photometry of the resolved brighter stars in this galaxy, 11 Cepheids with periods ranging between 12 and 90 days have been identified. The true distance modulus has been derived from the apparent distance moduli in g, r, and i. The distance modulus is 28.02 ± 0.18 based on the LMC period-luminosity (P-L) relation of Sandage, Reindl, & Tammann; this corresponds to a distance of 4.0 ± 0.3 Mpc. Using the P-L relation from Madore & Freedman, the distance modulus is 28.15 ± 0.18, which corresponds to a distance of 4.3 ± 0.4 Mpc. The reddening is calculated to be $E(g-r) = 0.06 \pm 0.08$ and $E(r-i) = 0.10 \pm 0.08$, again from the distance moduli μ_g, μ_r, and μ_i. In addition, 37 other variables have been detected, the majority of which have definite periods; they are probably all red long-period variables.

Key words: Cepheids — distance scale — galaxies: individual (NGC 4395) — stars: variables: other

On-line material: FITS file

1. INTRODUCTION

NGC 4395 ($\alpha = 12^h25^m48\overset{s}{.}92, \delta = +33°32^m48\overset{s}{.}3$; J2000) is a face-on spiral galaxy that is classified as Sd III–IV in the Carnegie Atlas of Galaxies (Sandage & Bedke 1994). It is located in the Ursa Major region called Group B4 by Kraan-Korteweg & Tammann (1979) and CVn I by de Vaucouleurs (1975).

NGC 4395 is a pure disk galaxy, i.e., it lacks a bulge (Filippenko & Ho 2003). It is also the nearest known Seyfert 1 galaxy and has a central black hole for which the total mass has recently been derived by Filippenko & Ho (2003). They estimate the mass of the central black hole to be $\sim 10^4$–$10^5\ M_\odot$, which is less massive than those found in other AGNs. All other galaxies with known supermassive black holes in their centers have bulges, which makes NGC 4395 a unique object.

Literature distance estimates range between 2.6 Mpc (Rowan-Robinson 1985) and 5.0 Mpc (Sandage & Bedke 1994). More recently, Karachentsev & Drozdovsky (1998) estimated the distance to NGC 4395 to be 4.2 ± 0.8 Mpc using the brightest blue stars. *Hubble Space Telescope* (*HST*) WFPC2 images have been used by two groups for the distance estimate. Karachentsev et al. (2003) obtained a tip of the red giant branch (TRGB) distance of 4.61 ± 0.57 Mpc; Minitti et al. (2004) derived a distance of 4.2 Mpc.

Repeated time-spaced images of NGC 4395 were obtained as part of a program begun by A. S. and J. G. H. in 1990 to determine Cepheid distances to several galaxies in the Local Group and beyond. The project was motivated in part by the desire to extend the range of ground-based Cepheid distances by employing the technological advantage of CCDs. While *HST* would surely vastly extend this range, we proceeded on the premise that relatively nearby objects that can be reached from the ground would not, and should not, occupy the precious and expensive resources of *HST*. Nevertheless, the cartography of the local universe continued to be an important problem.

Ground-based time-sequence observations are inevitably impeded by constraints from telescope scheduling, weather, and poor seeing. As a result, some of the objects (particularly the more distant and more demanding ones) selected in our initial program with the Kitt Peak 2.1 m telescope could not be completed. With the commissioning of the WIYN 3.5 m telescope (also on Kitt Peak), we got not only an increase in aperture, but also better image quality. In additional (and very importantly), while the NOAO queue observations were being conducted regularly, the necessary observations for the unfinished galaxies could be obtained efficiently.

While the time span to obtain the observations for the Cepheid discovery in NGC 4395 was a frustrating 8 years, a side benefit is that many long-period variables (LPVs) were also discovered for which periods could be determined. The literature lacks period, luminosity, and color information for LPVs, except in a very few galaxies (the Magellanic Clouds, M33), and consequently the empirical properties of LPVs

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and their dependence on metallicity (for instance) are very poorly known. We hope that the LPVs we have been able to catalog here for NGC 4395 is a step towards ameliorating this situation.

2. OBSERVATIONS

Images of NGC 4395 were taken using two different telescopes, the WIYN 3.5 m telescope and the 2.1 m KPNO telescope, both located on Kitt Peak. Repeated images were obtained over a period of 8 yr, from 1991 January 13 to 1999 January 13, in the g, r, and i passbands of the Thuan & Gunn (1976) system.

The detectors that we used are named TI-2, TIKA, and S2KB. TI-2, used on the 2.1 m, is a 796 × 796 CCD with a pixel size of 15 μm and a scale of 0″.2 per pixel. T1KA, also used on the 2.1 m, is a 1024 × 1024 CCD that has a pixel scale of 0″.305 per pixel and a field of view of 5′.2 × 5′.2. S2KB, used on the WIYN telescope, is a 2048 × 2048 CCD with 21 μm pixels, which provides a pixel scale of 0″.197 per pixel and a field of view of 6′.8 × 6′.8. Exposure times are between 500 and 1200 s. A journal of the observations is given in Table 1.

Primary observations were in the r-band, which was optimal for the color response of the early CCDs used, given both the color range of Cepheids and the region of the spectrum where the sky is sufficiently dark. Supplemental observations were made at four epochs in the g band and at two epochs in the i band, so that mean colors for the Cepheids could be derived and used to track extinction and reddening.

TABLE 1
JOURNAL OF OBSERVATIONS

Epoch (HJD)[a]	Exposure Time (s)	Filter	Seeing (arcsec)	Telescope	Instrument
48269.94............	4 × 900	r	1.0	KP 2.1 m	T1KA
48359.75............	3 × 900	r	1.0	KP 2.1 m	TI-2
48388.77............	3 × 1200	r	1.0	KP 2.1 m	TI-2
48622.95............	3 × 900	r	1.3	KP 2.1 m	T1KA
48622.95............	2 × 900	g	1.2	KP 2.1 m	T1KA
49064.83............	3 × 1200	r	1.0	KP 2.1 m	T1KA
49064.83............	1 × 1200	g	1.0	KP 2.1 m	T1KA
49076.86............	3 × 900	g	1.2	KP 2.1 m	T1KA
49417.87............	4 × 600	i	1.5	KP 2.1 m	T1KA
49421.84............	3 × 900	r	1.1	KP 2.1 m	T1KA
49427.87............	3 × 900	r	1.0	KP 2.1 m	T1KA
50189.89............	900	r	0.76	WIYN	S2KB
50190.91............	900	r	1.10	WIYN	S2KB
50191.71............	900	r	0.90	WIYN	S2KB
50192.73............	900	r	1.02	WIYN	S2KB
50213.83............	900	r	1.08	WIYN	S2KB
50216.88............	900	r	1.04	WIYN	S2KB
50226.64............	900	i	0.94	WIYN	S2KB
50226.69............	900	r	0.96	WIYN	S2KB
50489.92............	552	r	0.84	WIYN	S2KB
50509.77............	1200	r	0.74	WIYN	S2KB
50510.95............	1200	r	0.74	WIYN	S2KB
50511.79............	1200	r	0.96	WIYN	S2KB
50834.05............	900	r	0.72	WIYN	S2KB
50835.93............	1200	r	1.18	WIYN	S2KB
50837.02............	1200	r	0.72	WIYN	S2KB
50837.02............	1500	g	0.68	WIYN	S2KB
50867.92............	1200	r	0.92	WIYN	S2KB
50870.96............	1200	r	0.86	WIYN	S2KB
50872.94............	1200	r	0.88	WIYN	S2KB
50873.94............	1200	r	0.92	WIYN	S2KB
50875.93............	1200	r	0.68	WIYN	S2KB
50889.97............	1200	r	0.86	WIYN	S2KB
50894.75............	1200	r	0.78	WIYN	S2KB
50896.85............	1200	r	0.74	WIYN	S2KB
50932.79............	1200	r	0.86	WIYN	S2KB
50936.90............	1200	r	1.08	WIYN	S2KB
50965.71............	900	r	1.10	WIYN	S2KB
50987.67............	1200	r	0.86	WIYN	S2KB
50991.68............	1200	r	0.72	WIYN	S2KB
50996.69............	1200	r	1.06	WIYN	S2KB
51142.03............	1200	r	0.78	WIYN	S2KB
51171.99............	1200	r	0.92	WIYN	S2KB
51192.04............	1200	r	0.68	WIYN	S2KB

[a] HJD−2400000.

3. DATA PROCESSING

3.1. *Relative Photometry and Object Matching*

Multiple exposures obtained with WIYN S2KB were combined and used as a template FITS file. The reference image in *r* is shown in Figure 1. This image is also provided as a FITS file in the electronic version of this paper. Positions (*X* and *Y* in pixels) of objects given in various tables in this paper can be easily located on the electronic image. The published FITS image is also appointed with a ‘‘world coordinate system’’ (WCS) calibrated to J2000 coordinates on the sky.

The reference image is used as a position reference only. For visualization and ‘‘blinking’’ of images, the individual epoch images were mapped to the reference image. However, photometry (see below) for the individual epochs was always done on the untransformed original images, so that noise characteristics on the image remain untainted and also to avoid the seeing degradation from the transformation.

Relative photometry and object identification on the template images were obtained with the program DoPHOT (Schechter, Mateo, & Saha 1993). Subsequently, DoPHOT was run on each individual epoch. All objects identified in the individual epochs were matched to the template frame, and the coordinate transformations for the individual epochs were derived. The relative photometry at each epoch differs from that in the template frame by a constant magnitude offset. This offset is easily determined from the ensemble average of the difference between matched stars in the template and the individual epoch images. In order to avoid issues with bias at faint magnitudes and with bad outliers, this procedure is best done by inspecting a plot of the magnitude difference for each star versus the magnitude on the template image. This puts the relative magnitudes of all stars in all epochs in *r* on the same footing. A similar procedure was used for the *g* and *i* passbands as well. In practice, the template magnitudes in the three bands had already been adjusted to the true Thuan-Gunn system using the procedure described in § 3.2, so the final magnitudes for each epoch were also on the Thuan-Gunn system.

3.2. *Calibrating the Photometry*

Additional observations of NGC 4395, along with observations of standard stars, were taken with the S2KB detector on the WIYN 3.5 m telescope in the Thuan-Gunn *g*, *r*, and *i* bands on 2000 May 26 (UT). The images of NGC 4395 taken on this night were shallower exposures to ensure that some of the brighter isolated stars would not be saturated. A photometric solution was obtained for that night from the standard star observations. The error in the mean for the primary standard stars on this night is less than 0.01 mag in all three passbands.

Aperture photometry of several bright and relatively uncrowded stars in the field of NGC 4395 were obtained, and true

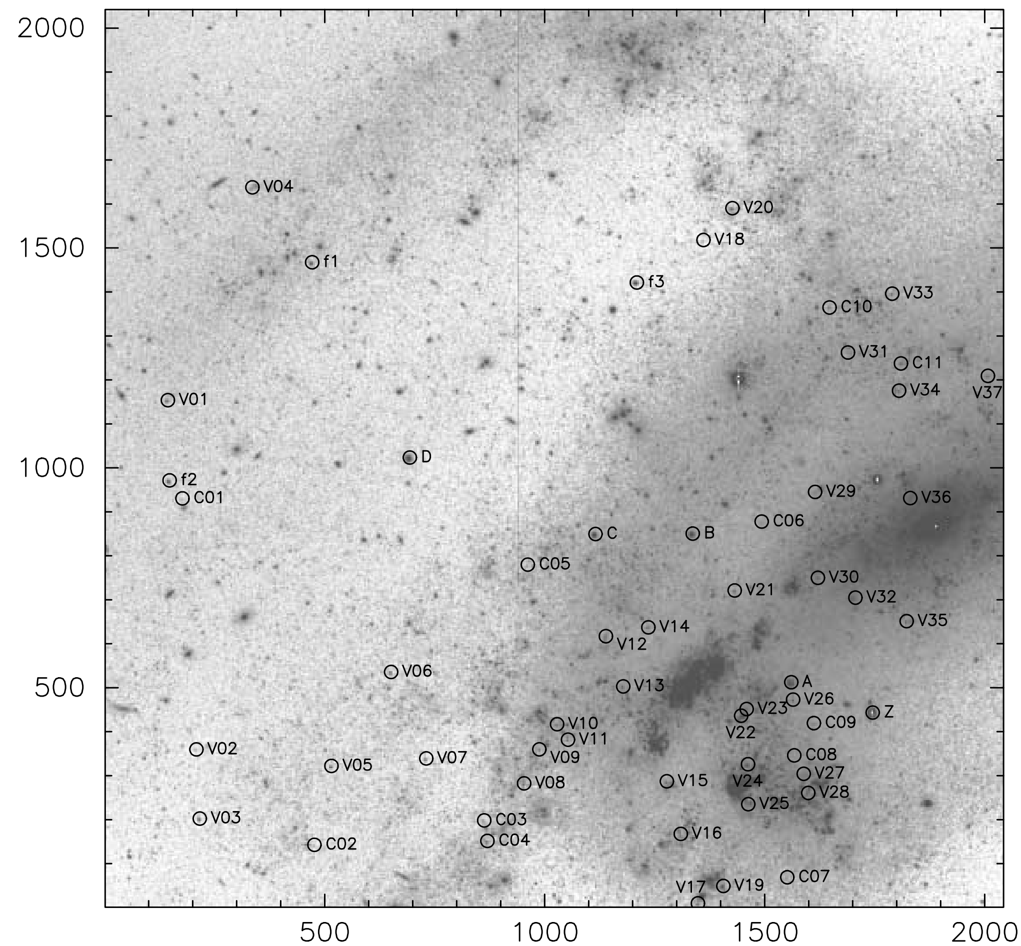

FIG. 1.—WIYN S2KB *r* image covering the field northeast of the center of NGC 4395, which is located on the right side close to the edge of the chip. North is up and east is to the left. Cepheids and other long-period variables are shown as open circles and labeled with C for Cepheids and V for other variables, followed by a number. The labels are the same as in Tables 5 and 6. The local standards that are presented in Table 2 are also shown as open circles and are labeled by the letters A, B, C, D, Z, f1, f2, and f3. [This figure is also available as a FITS file in the electronic edition of the Journal.]

TABLE 2
Local Standards

Identifier	X	Y	g	r	i	R.A. (J2000)	Decl. (J2000)
A	1561	513	17.59	16.97	16.76	12 25 54.2	+33 31 40
B	1337	851	18.69	18.13	17.93	12 25 57.5	33 32 47
C	1116	850	18.49	18.20	18.09	12 26 01.2	33 32 48
D	694	1024	18.54	17.41	16.36	12 26 08.1	33 33 24
Z	1746	444	16.03	15.81	15.73	12 25 51.2	33 31 26
f1	472	1468	21.77	20.55	20.10	12 26 11.2	33 34 51
f2	148	972	20.94	20.12	19.78	12 26 17.3	33 33 16
f3	1210	1422	20.53	19.39	18.00	12 25 58.9	33 34 39

Note.—Units of right ascension are hours, minutes, and seconds, and units of declination are degrees, arcminutes, and arcseconds.

magnitudes were derived using the photometric solution for that night. Sky subtraction in all cases was done using the "growth curve" method. Thus, local standards in *g*, *r*, and *i* were established in the field of NGC 4395. These local standards are identified by the letters A, B, C, D, and Z and are shown in Figure 1. Their magnitudes are listed in Table 2, along with their *X* and *Y* positions on the template image and their coordinates on the sky. The standard rms error of measurement for each star is less than 0.015 mag. In addition, three relatively fainter stars were also calibrated as local standards, but using PSF fitting and aperture correction. These stars are designated as f1, f2, and f3, and are also shown in Figure 1 with magnitudes listed in Table 2. For these three fainter objects, the standard rms error is $\sim$0.03 mag per star. A conservative estimate is that the magnitude scales and colors are calibrated to within a systematic error not exceeding 0.05 mag.

Armed with this calibration, the relative photometry described in § 3.1 is adjusted to the true Thuan-Gunn system by comparing the relative photometry of an ensemble of several thousand stars from any epoch or template with stars found in common on the images taken on the calibrating night.

4. IDENTIFICATION OF THE VARIABLE STARS

4.1. *Detection of Variable Stars*

Given photometry and photometric errors in all epochs, each object was examined for variability. The method for identifying variable stars is described in Saha & Hoessel (1990) and is not repeated here. The requirements for variability have been improved since Saha & Hoessel (1990), from experience with similar data. The individual magnitudes measured in the various epochs and their individual errors estimates for each of the Cepheids in *g*, *r*, and *i* are tabulated in Table 3; the individual magnitudes and errors of the other variable stars in *g*, *r*, and *i* are tabulated in Table 4.

If P is the period of a supposed variable star, m_i the measured magnitude at the *i*th epoch, and $\overline{m}$ the average over the n values of m_i, and if the values for m_i are arranged in increasing order of phase, then Θ is defined as

$$\Theta(P) = \frac{\sum_{i=1}^{n} (m_{i+1} - m_i)^2}{\sum_{i=1}^{n} (m_i - \overline{m})^2}. \quad (1)$$

A minimum in the spectrum of Θ indicates a possible period. The reduced χ^2 is given by

$$\chi^2_\nu = \sum_i \frac{(m - \overline{m})^2}{\nu \sigma_i^2}, \quad (2)$$

where σ_i is the rms error and $\nu = n - 1$. The variable Λ (Lafler & Kinman 1965; Saha & Hoessel 1990) is related to Θ in that Λ scales as Θ^{-1}. An object is considered to be variable if the χ^2 probability is more than 0.99 and Λ is greater than 3.5. Furthermore, an object with a reduced χ^2 more than 100 is always flagged as a possible variable. The requirements and changes from Saha & Hoessel (1990) are described in more detail in Saha, Claver, & Hoessel (2002).

Sixty-four out of a total of 25,155 stars in the template *r* frame are flagged as possible variables. These candidates are examined for periodicity using the method of Lafler & Kinman (1965). The period range from 1 to 3000 days has been used to probe possible variability. Only data in the *r* band have been used for the variable search, because only four epochs in the *g* and two in the *i* band were taken, which is insufficient to derive a period. The objects that are likely to be Cepheids are presented in Table 5; the remaining other variables are listed in Table 6. As a double check on the variability, all candidates have been visually blinked and compared with nonvariable stars. The periods are given in column (2). The periods for some variables in Table 6 are uncertain, with uncertainties ranging from a few to a few hundred days. In such cases, we present our best guess in the table.

The mean magnitudes in *r* are phase-weighted intensity averaged means and were calculated using the equation

$$\langle r \rangle = -2.5 \log_{10} \sum_{i=1}^{n} 0.5(\phi_{i+1} - \phi_{i-1}) 10^{-0.4 r_i}, \quad (3)$$

where n is the number of observations, r_i the magnitude, and ϕ_i the phase of the *i*th observation in order of increasing phase. Intensity-weighted magnitudes can be biased by missing measurements—biased in that it is more likely to detect the variable star at a brighter rather than at a fainter phase. The phase-weighted intensity mean gives isolated points more weight than closely spaced ones, which makes it superior to a straight intensity mean. The method of using well-sampled light curves to predict corrections to small samplings at another wavelength in order to predict their mean magnitudes originates with Freedman (1988). Here we use a similar method described in Labhardt, Sandage, & Tammann (1997). The mean *g* magnitudes in Table 5 are calculated using the information on the shape and the amplitude of the complete light curve in one filter, and the typical phase shift between two filters is used to derive a value of $\langle g \rangle$ from each *g* measurement. Labhardt et al. (1997) present transformations for the filters *B*, *V*, *R*, and *I*. Therefore, the empirical correction

TABLE 3

Photometry of the Cepheids: Magnitudes and Error Estimates

HJD	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
					r						
2448269.9406............	. . .	23.81 ± ± 0.16	. . .	21.30 ± 0.03	23.98 ± 0.18	23.15 ± 0.10	23.23 ± 0.08	. . .	22.15 ± 0.04	22.05 ± 0.04	21.51 ± 0.02
2448359.7523............	. . .	. . .	. . .	. . .	23.27 ± 0.10	. . .	. . .	. . .	. . .	. . .	. . .
2448388.7788............	. . .	. . .	. . .	. . .	23.18 ± 0.12	. . .	. . .	. . .	. . .	. . .	. . .
2448622.9552............	. . .	. . .	23.86 ± 0.18	21.30 ± 0.02	23.22 ± 0.10	22.83 ± 0.25	22.92 ± 0.09	. . .	22.27 ± 0.04	22.61 ± 0.06	21.46 ± 0.04
2449064.8339............	. . .	. . .	. . .	21.39 ± 0.02	22.86 ± 0.07	22.93 ± 0.08	22.62 ± 0.08	. . .	22.47 ± 0.09	22.05 ± 0.04	21.46 ± 0.04
2449421.8408............	. . .	24.12 ± 0.14	24.30 ± 0.17	21.39 ± 0.02	. . .	22.86 ± 0.05	22.31 ± 0.03	24.05 ± 0.19	22.14 ± 0.04	22.52 ± 0.04	. . .
2449427.8788............	. . .	. . .	23.07 ± 0.06	21.41 ± 0.01	22.85 ± 0.05	23.06 ± 0.05	22.58 ± 0.04	. . .	22.14 ± 0.04	22.12 ± 0.03	21.43 ± 0.01
2450189.8905............	23.38 ± 0.08	24.03 ± 0.11	23.77 ± 0.10	21.30 ± 0.02	22.83 ± 0.04	23.49 ± 0.08	23.10 ± 0.06	24.25 ± 0.19	22.35 ± 0.04	. . .	. . .
2450190.9141............	22.95 ± 0.09	. . .	. . .	21.30 ± 0.02	22.95 ± 0.09	23.31 ± 0.13	23.05 ± 0.09	. . .	22.26 ± 0.05	22.34 ± 0.05	21.73 ± 0.03
2450191.7105............	23.23 ± 0.06	24.21 ± 0.13	23.58 ± 0.09	21.30 ± 0.01	23.11 ± 0.06	23.13 ± 0.06	22.93 ± 0.05	24.22 ± 0.18	22.26 ± 0.04	22.26 ± 0.03	21.75 ± 0.02
2450192.7317............	23.11 ± 0.10	24.05 ± 0.18	. . .	21.30 ± 0.02	23.12 ± 0.09	22.97 ± 0.08	22.97 ± 0.09	. . .	22.30 ± 0.06	22.27 ± 0.05	21.84 ± 0.03
2450213.8314............	. . .	. . .	23.56 ± 0.12	21.51 ± 0.02	22.86 ± 0.06	23.44 ± 0.11	22.68 ± 0.08	. . .	22.59 ± 0.05	22.49 ± 0.04	21.49 ± 0.02
2450216.8806............	. . .	. . .	. . .	21.46 ± 0.02	22.74 ± 0.07	23.00 ± 0.08	22.76 ± 0.06	. . .	22.59 ± 0.06	22.38 ± 0.05	21.42 ± 0.02
2450226.6969............	23.81 ± 0.13	23.77 ± 0.13	23.30 ± 0.08	21.47 ± 0.02	23.41 ± 0.15	23.13 ± 0.08	23.11 ± 0.07	. . .	22.15 ± 0.04	21.99 ± 0.03	21.35 ± 0.02
2450489.9295............	23.11 ± 0.09	. . .	24.09 ± 0.22	21.39 ± 0.02	23.19 ± 0.09	23.35 ± 0.12	22.61 ± 0.06	24.01 ± 0.22	22.43 ± 0.05	22.08 ± 0.04	21.48 ± 0.02
2450509.7780............	23.18 ± 0.04	23.51 ± 0.07	23.43 ± 0.07	21.45 ± 0.01	23.05 ± 0.05	23.05 ± 0.05	23.50 ± 0.08	24.46 ± 0.19	22.49 ± 0.04	22.27 ± 0.02	21.50 ± 0.02
2450510.9513............	23.30 ± 0.06	23.67 ± 0.08	23.17 ± 0.06	21.52 ± 0.02	23.21 ± 0.05	23.03 ± 0.05	23.00 ± 0.05	23.65 ± 0.10	22.38 ± 0.04	22.26 ± 0.03	21.39 ± 0.01
2450511.7910............	23.17 ± 0.09	23.84 ± 0.14	23.21 ± 0.08	21.41 ± 0.02	23.11 ± 0.06	23.10 ± 0.07	22.59 ± 0.04	23.72 ± 0.16	22.30 ± 0.04	22.38 ± 0.04	21.51 ± 0.02
2450834.0550............	23.78 ± 0.16	23.45 ± 0.11	23.75 ± 0.14	21.31 ± 0.02	23.58 ± 0.12	23.57 ± 0.12	22.97 ± 0.07	. . .	22.38 ± 0.04	22.42 ± 0.04	21.65 ± 0.02
2450835.9316............	23.76 ± 0.16	23.75 ± 0.15	23.97 ± 0.22	21.35 ± 0.02	23.59 ± 0.12	23.18 ± 0.08	22.94 ± 0.08	. . .	22.38 ± 0.04	22.45 ± 0.05	21.56 ± 0.05
2450837.0266............	23.77 ± 0.08	23.72 ± 0.07	23.83 ± 0.08	21.31 ± 0.01	23.41 ± 0.05	22.99 ± 0.04	23.12 ± 0.04	24.53 ± 0.18	22.47 ± 0.04	22.55 ± 0.03	. . .
2450867.9278............	23.18 ± 0.08	23.85 ± 0.14	23.68 ± 0.12	21.49 ± 0.02	23.62 ± 0.11	22.88 ± 0.06	23.09 ± 0.07	23.98 ± 0.17	22.09 ± 0.04	22.10 ± 0.03	21.45 ± 0.02
2450870.9631............	23.44 ± 0.06	23.80 ± 0.09	23.47 ± 0.07	21.45 ± 0.01	22.93 ± 0.05	22.91 ± 0.04	23.24 ± 0.06	. . .	22.12 ± 0.04	22.07 ± 0.02	21.39 ± 0.01
2450872.9496............	23.65 ± 0.09	23.57 ± 0.08	23.11 ± 0.05	21.39 ± 0.01	22.88 ± 0.05	23.02 ± 0.06	23.34 ± 0.06	24.57 ± 0.23	22.09 ± 0.02	22.15 ± 0.02	21.48 ± 0.01
2450873.9432............	23.56 ± 0.10	23.40 ± 0.09	23.30 ± 0.11	21.40 ± 0.02	22.90 ± 0.05	23.07 ± 0.07	23.24 ± 0.09	23.89 ± 0.15	22.10 ± 0.04	22.16 ± 0.03	21.46 ± 0.01
2450875.9306............	23.88 ± 0.08	23.80 ± 0.06	23.46 ± 0.05	21.35 ± 0.01	23.03 ± 0.04	23.22 ± 0.04	22.35 ± 0.03	23.83 ± 0.09	22.17 ± 0.02	22.18 ± 0.02	21.45 ± 0.01
2450889.9782............	. . .	24.18 ± 0.23	23.40 ± 0.12	21.26 ± 0.02	23.52 ± 0.13	22.90 ± 0.07	23.05 ± 0.09	. . .	22.38 ± 0.05	22.49 ± 0.06	21.57 ± 0.03
2450894.7555............	23.85 ± 0.13	23.89 ± 0.15	. . .	21.24 ± 0.02	23.28 ± 0.09	22.90 ± 0.07	23.31 ± 0.09	. . .	22.40 ± 0.04	22.59 ± 0.05	21.63 ± 0.02
2450896.8575............	23.84 ± 0.08	23.71 ± 0.06	23.84 ± 0.07	21.23 ± 0.01	22.93 ± 0.04	22.89 ± 0.04	23.11 ± 0.05	24.34 ± 0.17	22.44 ± 0.04	22.61 ± 0.03	21.64 ± 0.01
2450932.7955............	23.88 ± 0.12	24.19 ± 0.16	23.06 ± 0.07	21.44 ± 0.02	23.09 ± 0.06	23.45 ± 0.10	22.28 ± 0.03	. . .	22.10 ± 0.04	22.27 ± 0.04	21.38 ± 0.02
2450936.9023............	23.24 ± 0.11	. . .	23.58 ± 0.17	21.40 ± 0.03	23.48 ± 0.14	23.55 ± 0.17	22.50 ± 0.07	23.75 ± 0.20	22.24 ± 0.05	22.38 ± 0.06	21.42 ± 0.03
2450965.7179............	23.52 ± 0.12	. . .	23.40 ± 0.11	21.24 ± 0.02	23.67 ± 0.14	23.20 ± 0.13	22.52 ± 0.05	. . .	22.55 ± 0.05	22.03 ± 0.03	21.47 ± 0.02
2450987.6717............	23.78 ± 0.11	23.77 ± 0.09	23.92 ± 0.12	21.38 ± 0.01	23.26 ± 0.05	23.56 ± 0.09	22.57 ± 0.04	24.33 ± 0.17	22.18 ± 0.04	22.38 ± 0.03	21.80 ± 0.02
2450991.6879............	23.69 ± 0.06	23.72 ± 0.06	23.36 ± 0.05	21.41 ± 0.01	23.44 ± 0.05	23.09 ± 0.04	22.53 ± 0.03	24.57 ± 0.15	22.18 ± 0.02	22.38 ± 0.02	21.71 ± 0.01
2450996.6918............	23.30 ± 0.14	. . .	23.34 ± 0.14	21.49 ± 0.03	23.75 ± 0.20	22.93 ± 0.09	22.71 ± 0.07	23.67 ± 0.20	22.14 ± 0.04	22.43 ± 0.06	21.72 ± 0.03

TABLE 3—*Continued*

HJD	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
2451142.0300............	23.32 ± 0.06	24.06 ± 0.11	23.43 ± 0.06	21.47 ± 0.01	23.11 ± 0.05	23.42 ± 0.07	22.97 ± 0.05	23.84 ± 0.11	22.61 ± 0.04	22.27 ± 0.03	21.53 ± 0.01
2451171.9956............	23.59 ± 0.10	23.57 ± 0.08	23.38 ± 0.06	21.26 ± 0.01	23.30 ± 0.06	23.16 ± 0.06	23.11 ± 0.06	24.40 ± 0.18	22.22 ± 0.04	22.23 ± 0.03	21.80 ± 0.02
2451192.0452............	23.82 ± 0.06	24.18 ± 0.09	23.36 ± 0.05	21.30 ± 0.01	23.12 ± 0.04	23.56 ± 0.05	22.58 ± 0.03	24.05 ± 0.09	22.59 ± 0.04	22.17 ± 0.02	21.66 ± 0.01
						g					
2448622.9572............	...	...	...	21.45 ± 0.05	23.52 ± 0.15	22.72 ± 0.09	23.09 ± 0.11	...	22.61 ± 0.09	23.15 ± 0.11	21.91 ± 0.05
2449064.8359............	...	...	...	21.60 ± 0.05	...	...	22.78 ± 0.11	...	23.18 ± 0.19	22.41 ± 0.08	...
2449076.8680............	...	...	23.89 ± 0.15	21.78 ± 0.05	...	24.01 ± 0.16	23.82 ± 0.13	...	22.50 ± 0.05	22.72 ± 0.05	21.93 ± 0.05
2450837.0286............	24.19 ± 0.12	24.16 ± 0.12	24.59 ± 0.18	21.67 ± 0.05	24.01 ± 0.10	23.16 ± 0.05	23.66 ± 0.07	24.21 ± 0.13	23.15 ± 0.05	23.15 ± 0.05	22.18 ± 0.05
						i					
2449417.8790............	...	...	22.91 ± 0.13	21.11 ± 0.05	...	22.81 ± 0.14	23.33 ± 0.18	...	22.11 ± 0.08	22.59 ± 0.09	21.40 ± 0.05
2450226.6444............	23.58 ± 0.19	23.50 ± 0.18	23.26 ± 0.14	21.45 ± 0.05	23.24 ± 0.16	22.83 ± 0.09	23.33 ± 0.16	...	22.15 ± 0.05	22.05 ± 0.05	21.40 ± 0.05

function between V and R have been transformed to g and r using synthetic transformations and amplitude ratios.

4.2. *Identification of Cepheid Variables and Period Determination*

The light curves of all possible variable stars with reasonable values of Θ were individually inspected by eye in all three passbands. However, in g and i, there is sometimes only one, or even zero, photometric measurements, which makes the term ''light curve'' meaningless. In such cases, the color estimate is rather uncertain. The light curves of the Cepheid candidates are shown in Figure 2.

The quality of the light curves in r and g, the phase coherence between the r, g, and i light curves, the shape of the spectrum of Θ, and the g–r and r–i colors have been used as the selection criteria to consider whether a variable star is a Cepheid or not.

Since there are very few epochs with measurements in the g band, the mean $\langle g\rangle$ magnitudes are calculated using the method of Labhardt et al. (1997), with an equation of the form

$$\langle g\rangle = g(\phi_r) + [\langle r\rangle - r(\phi_r)] + \Delta r C_{r\rightarrow g}(\phi), \tag{4}$$

where Δr is the r amplitude, $\langle r\rangle$ is the phase-weighted mean r magnitude, ϕ is the phase of the light curve, and $C_{r\rightarrow g}(\phi)$ is the empirical function for the transformation between r and g magnitudes. Labhardt et al. (1997) listed values of $C(\phi)$ for B, V, R, and I mags, from which we can derive $C_{r\rightarrow g}(\phi)$, using the equation

$$C_{r\rightarrow g}(\phi) = 1.722 C_{V\rightarrow R}(\phi) + 0.655 C_{V\rightarrow B}(\phi). \tag{5}$$

$C_{V\rightarrow R}(\phi)$ are the correction values for $V \rightarrow R$ magnitudes, $C_{V\rightarrow B}(\phi)$ are the values for $V \rightarrow B$ magnitudes and are listed in Labhardt et al. (1997). The derivation of the above equation uses color transformations discussed in § 5.

The mean of the individual $\langle g\rangle$ magnitudes yields the adopted value of $\langle g\rangle$ and its error. The i magnitudes are calculated from the r–i color at the i phase and the mean $\langle r\rangle$ values (i.e., setting $C_{r\rightarrow i}(\phi)$ to zero).

In total, we identified 11 Cepheids, which are listed in Table 5. Column (1) gives the designation of the Cepheid, column (2) the period, columns (3)–(8) give the mean magnitudes and errors in g, r, and i, and columns (9)–(10) their position on the template r image.

As noted before, all variable stars have been visually blinked. Nine of the eleven Cepheids in Table 5 blink; it is hard to tell for Cepheids C4 and C11 because of their small amplitudes. Apart from these eleven candidates, no further Cepheid candidates have been accepted.

4.3. *Long-Period Variables*

The remaining candidates, which have not been classified as Cepheid variables, are listed in Table 6. We found 37 variables besides the 11 Cepheids. Again, column (1) gives the designation of the variable, column (2) the period, columns (3)–(5) give the mean magnitudes in g, r, and i, column (6) the r peak-to-peak amplitude, and columns (7)–(8) their position on the template r image. The r amplitude is estimated from the peak-to-peak magnitude from their smoothed light curves rather than from the difference of minimum and maximum measured magnitudes. All variables have been visually inspected by blinking pairs of images for variability, i.e., images near maximum brightness were compared against images near minimum brightness. All candidates in Table 6 vary intrinsically in brightness. Sixteen variables have an uncertain period; in these cases, the best guess for the period estimate is presented. Their light curves are presented in Figure 3.

The variables in Table 6 have g–r colors between 0.5 and 1.6 and r–i colors of about 0.2 to 1.2, which indicates that these variables are red giants. No luminous blue variables (LBVs) or Hubble-Sandage variables (Hubble & Sandage 1953) have been identified. A detailed discussion of the LPVs is taken up in § 7.2 after we have derived the distance modulus.

5. THE CEPHEID PERIOD-LUMINOSITY RELATION

5.1. *The Period-Luminosity Relation Using the Kent Transformations*

The transformation equations from the Johnson to the Thuan-Gunn filter system given by Kent (1985) are

$$V = g - 0.03 - 0.42(g - r), \tag{6}$$

$$R = r - 0.51 - 0.15(g - r). \tag{7}$$

For the i band, the Wade et al. (1979) transformations have been used. These transformations are valid for a color range typical for Cepheids given by

$$i = 0.999I + 0.690 + 0.419(R - I). \tag{8}$$

The period-luminosity (P-L) relation in *BVRI* from Madore & Freedman (1991) can be transformed using the relations of Kent (1985) and Wade et al. (1979) to

$$M_g = -2.62(\log P - 1) - 4.08, \tag{9}$$

$$M_r = -2.91(\log P - 1) - 4.04, \tag{10}$$

$$M_i = -3.00(\log P - 1) - 4.06, \tag{11}$$

as derived in Hoessel et al. (1994).

5.2. *The Period-Luminosity Relation Using Synthetic Transformations*

Transformations that better reflect the spectral energy distributions (SEDs) of supergiants with temperatures spanning those of Cepheids can be obtained synthetically. The observed SEDs of several Thuan-Gunn standard stars spanning a wide range of color were integrated for model bandpasses of g, r, and i. The transformation to convert the resulting ''instrumental'' magnitudes to true magnitudes in the Thuan-Gunn system was derived. The same was done for the *BVRI* Johnson-Cousins-Landolt system.

Once we could accurately reproduce the stellar photometry from the stellar spectra, we computed synthetic magnitudes in both filter systems, Johnson-Cousins-Landolt and Thuan-Gunn, from synthetic stellar spectra (Kurucz 2003).[4] Limiting this work to stars with temperatures and gravities within, and slightly beyond, the range of Cepheids, we computed the

[4] Available at http://kurucz.harvard.edu/.

TABLE 4

PHOTOMETRY OF THE VARIABLE STARS: MAGNITUDES AND ERROR ESTIMATES

HJD	V1	V2	V3	V4	V5	V6	V7	V8
				r				
2448269.9406............	...	...	...	...	20.64 ± 0.02	21.47 ± 0.02	21.88 ± 0.03	21.59 ± 0.03
2448359.7523............	...	...	...	...	...	21.65 ± 0.02	...	...
2448388.7788............	...	...	...	...	...	...	...	...
2448622.9552............	...	...	...	...	...	...	22.43 ± 0.06	21.61 ± 0.03
2449064.8339............	...	...	...	...	...	21.60 ± 0.03	22.07 ± 0.04	21.89 ± 0.04
2449421.8408............	...	...	...	...	20.55 ± 0.01	21.26 ± 0.01	22.39 ± 0.03	21.42 ± 0.01
2449427.8788............	...	...	...	...	...	21.26 ± 0.02	22.33 ± 0.03	21.39 ± 0.01
2450189.8905............	21.95 ± 0.02	...	20.68 ± 0.01	22.97 ± 0.05	21.05 ± 0.01	21.35 ± 0.01	22.41 ± 0.04	21.67 ± 0.02
2450190.9141............	22.01 ± 0.04	23.58 ± 0.14	20.74 ± 0.01	22.88 ± 0.09	21.07 ± 0.02	21.48 ± 0.02	22.24 ± 0.05	21.72 ± 0.03
2450191.7105............	22.05 ± 0.02	23.77 ± 0.10	20.70 ± 0.01	22.99 ± 0.05	21.13 ± 0.01	21.48 ± 0.02	22.46 ± 0.03	21.70 ± 0.02
2450192.7317............	21.95 ± 0.04	...	20.73 ± 0.01	22.95 ± 0.10	21.12 ± 0.01	21.48 ± 0.02	22.38 ± 0.05	21.69 ± 0.03
2450213.8314............	...	...	...	...	...	...	...	21.80 ± 0.03
2450216.8806............	...	...	...	...	...	...	...	21.77 ± 0.03
2450226.6969............	21.52 ± 0.02	23.39 ± 0.09	20.94 ± 0.01	22.99 ± 0.07	21.31 ± 0.02	21.59 ± 0.02	22.08 ± 0.03	21.81 ± 0.02
2450489.9295............	21.57 ± 0.02	23.11 ± 0.08	21.68 ± 0.03	23.41 ± 0.10	21.17 ± 0.02	22.06 ± 0.03	22.14 ± 0.03	21.60 ± 0.02
2450509.7780............	21.61 ± 0.01	23.32 ± 0.07	21.60 ± 0.01	...	21.20 ± 0.01	22.12 ± 0.02	22.07 ± 0.02	21.59 ± 0.01
2450510.9513............	21.66 ± 0.02	...	21.63 ± 0.01	23.53 ± 0.09	21.28 ± 0.01	22.30 ± 0.03	22.22 ± 0.03	21.69 ± 0.02
2450511.7910............	21.58 ± 0.02	23.30 ± 0.08	21.63 ± 0.02	...	21.22 ± 0.01	22.07 ± 0.03	22.00 ± 0.03	21.59 ± 0.02
2450834.0550............	21.56 ± 0.02	23.35 ± 0.10	20.92 ± 0.01	23.49 ± 0.11	20.71 ± 0.01	21.68 ± 0.02	22.46 ± 0.05	21.35 ± 0.02
2450835.9316............	21.56 ± 0.02	23.24 ± 0.10	20.89 ± 0.01	23.71 ± 0.15	20.74 ± 0.01	21.75 ± 0.03	22.44 ± 0.05	21.43 ± 0.02
2450837.0266............	21.48 ± 0.01	23.44 ± 0.05	20.94 ± 0.01	23.62 ± 0.06	20.63 ± 0.01	21.53 ± 0.01	22.44 ± 0.03	21.36 ± 0.01
2450867.9278............	21.47 ± 0.01	23.08 ± 0.07	21.13 ± 0.01	23.67 ± 0.13	20.75 ± 0.01	21.30 ± 0.01	22.45 ± 0.03	21.34 ± 0.02
2450870.9631............	21.49 ± 0.01	23.14 ± 0.05	21.17 ± 0.01	23.65 ± 0.10	20.80 ± 0.01	21.38 ± 0.01	22.55 ± 0.04	21.42 ± 0.01
2450872.9496............	21.44 ± 0.01	23.05 ± 0.04	21.19 ± 0.01	23.48 ± 0.07	20.78 ± 0.01	21.31 ± 0.01	22.42 ± 0.03	21.38 ± 0.01
2450873.9432............	21.46 ± 0.01	23.08 ± 0.05	21.23 ± 0.01	23.52 ± 0.10	20.80 ± 0.01	21.31 ± 0.01	22.43 ± 0.03	21.35 ± 0.01
2450875.9306............	21.46 ± 0.01	23.10 ± 0.04	21.18 ± 0.01	...	20.80 ± 0.01	21.31 ± 0.01	22.46 ± 0.02	21.38 ± 0.01
2450889.9782............	...	23.09 ± 0.10	21.34 ± 0.02	...	20.86 ± 0.01	21.24 ± 0.02	22.25 ± 0.05	21.40 ± 0.02
2450894.7555............	21.42 ± 0.02	23.16 ± 0.08	21.35 ± 0.02	23.48 ± 0.12	20.88 ± 0.01	21.25 ± 0.02	22.25 ± 0.03	21.42 ± 0.02
2450896.8575............	21.51 ± 0.01	23.25 ± 0.04	21.36 ± 0.01	...	20.90 ± 0.01	21.31 ± 0.01	22.31 ± 0.03	21.42 ± 0.01
2450932.7955............	21.62 ± 0.02	23.42 ± 0.08	21.65 ± 0.02	22.92 ± 0.06	21.01 ± 0.02	21.36 ± 0.02	22.17 ± 0.03	21.55 ± 0.02
2450936.9023............	21.55 ± 0.04	23.18 ± 0.11	21.67 ± 0.03	22.90 ± 0.10	20.95 ± 0.02	21.25 ± 0.02	22.02 ± 0.05	21.48 ± 0.02
2450965.7179............	21.59 ± 0.02	23.55 ± 0.11	21.88 ± 0.03	...	21.05 ± 0.01	21.48 ± 0.02	22.03 ± 0.03	21.52 ± 0.02
2450987.6717............	21.46 ± 0.01	23.77 ± 0.09	22.00 ± 0.02	23.31 ± 0.08	21.00 ± 0.01	21.44 ± 0.01	22.00 ± 0.02	21.50 ± 0.01
2450991.6879............	21.49 ± 0.01	23.90 ± 0.08	22.01 ± 0.01	23.76 ± 0.07	21.01 ± 0.01	21.49 ± 0.01	22.05 ± 0.01	21.48 ± 0.01
2450996.6918............	21.49 ± 0.04	24.07 ± 0.25	22.03 ± 0.04	23.27 ± 0.14	21.05 ± 0.02	21.53 ± 0.03	22.01 ± 0.05	21.50 ± 0.02
2451142.0300............	22.16 ± 0.02	23.59 ± 0.07	22.22 ± 0.02	...	21.18 ± 0.01	22.38 ± 0.02	21.99 ± 0.02	21.85 ± 0.02
2451171.9956............	22.27 ± 0.04	...	22.08 ± 0.02	23.45 ± 0.09	21.27 ± 0.01	22.43 ± 0.03	22.19 ± 0.04	21.72 ± 0.02
2451192.0452............	22.36 ± 0.02	23.72 ± 0.05	21.84 ± 0.01	...	21.35 ± 0.01	22.44 ± 0.02	22.23 ± 0.02	21.53 ± 0.01
				g				
2448622.9572............	...	...	...	...	...	...	23.26 ± 0.10	...
2449064.8359............	...	...	...	...	...	22.84 ± 0.11	...	...
2449076.8680............	...	...	...	...	...	...	...	23.30 ± 0.09
2450837.0286............	22.97 ± 0.05	24.83 ± 0.18	22.48 ± 0.05	24.67 ± 0.17	22.17 ± 0.05	22.85 ± 0.05	23.58 ± 0.06	22.82 ± 0.05
				i				
2449417.8790............	...	...	...	...	19.93 ± 0.05	20.64 ± 0.05	21.59 ± 0.05	20.81 ± 0.05
2450226.6444............	20.74 ± 0.05	22.20 ± 0.06	20.19 ± 0.05	21.92 ± 0.05	20.27 ± 0.05	20.88 ± 0.05	21.69 ± 0.05	20.96 ± 0.05

TABLE 4—*Continued*

V9	V10	V11	V12	V13	V14	V15	V16	V17
				r				
22.27 ± 0.05	21.86 ± 0.03	21.68 ± 0.02	20.99 ± 0.02	21.69 ± 0.02	21.69 ± 0.03	21.67 ± 0.03	23.38 ± 0.16	...
...	...	...	20.76 ± 0.01	21.51 ± 0.03	21.80 ± 0.03	...	...	...
...	...	...	20.72 ± 0.01	21.38 ± 0.03	21.97 ± 0.04	...	...	...
22.23 ± 0.05	21.88 ± 0.03	22.24 ± 0.04	20.90 ± 0.02	21.43 ± 0.02	21.23 ± 0.02	20.76 ± 0.01	23.30 ± 0.13	22.21 ± 0.05
22.06 ± 0.04	22.20 ± 0.04	...	20.67 ± 0.01	21.05 ± 0.02	21.80 ± 0.04	21.59 ± 0.03	...	21.94 ± 0.04
21.84 ± 0.02	21.97 ± 0.03	22.20 ± 0.03	20.80 ± 0.01	21.57 ± 0.02	21.59 ± 0.02	20.85 ± 0.01	23.32 ± 0.09	22.19 ± 0.04
21.85 ± 0.02	22.06 ± 0.03	22.05 ± 0.03	20.78 ± 0.01	21.55 ± 0.02	21.61 ± 0.02	20.83 ± 0.01	23.17 ± 0.06	...
22.51 ± 0.04	21.75 ± 0.02	22.08 ± 0.03	21.30 ± 0.01	21.16 ± 0.01	22.05 ± 0.02	21.60 ± 0.02	23.34 ± 0.07	21.69 ± 0.02
22.58 ± 0.09	21.73 ± 0.03	22.18 ± 0.04	21.39 ± 0.02	21.18 ± 0.02	22.06 ± 0.05	21.63 ± 0.04	23.50 ± 0.18	21.63 ± 0.04
22.58 ± 0.04	21.85 ± 0.02	22.16 ± 0.03	21.43 ± 0.02	21.23 ± 0.01	22.11 ± 0.03	21.58 ± 0.02	23.05 ± 0.05	21.49 ± 0.02
22.61 ± 0.05	21.80 ± 0.03	22.16 ± 0.05	21.42 ± 0.02	21.26 ± 0.02	22.05 ± 0.04	21.64 ± 0.04	23.36 ± 0.12	21.58 ± 0.05
22.48 ± 0.08	21.81 ± 0.03	22.15 ± 0.04	21.26 ± 0.02	21.23 ± 0.02	21.88 ± 0.03	21.56 ± 0.03	22.80 ± 0.07	21.55 ± 0.02
22.35 ± 0.04	21.76 ± 0.03	22.19 ± 0.04	21.28 ± 0.02	21.18 ± 0.02	21.93 ± 0.04	21.55 ± 0.03	22.71 ± 0.09	21.53 ± 0.04
22.47 ± 0.04	21.93 ± 0.03	22.26 ± 0.04	21.33 ± 0.02	21.26 ± 0.02	21.99 ± 0.03	21.55 ± 0.03	...	21.46 ± 0.02
22.14 ± 0.04	21.91 ± 0.04	22.47 ± 0.08	21.38 ± 0.02	21.67 ± 0.03	21.74 ± 0.03	21.32 ± 0.03	23.47 ± 0.12	22.28 ± 0.04
22.28 ± 0.02	21.91 ± 0.02	22.43 ± 0.03	21.20 ± 0.01	21.71 ± 0.02	21.72 ± 0.02	21.35 ± 0.02	23.61 ± 0.07	22.18 ± 0.04
22.41 ± 0.04	22.07 ± 0.03	22.62 ± 0.04	21.33 ± 0.02	21.82 ± 0.02	21.81 ± 0.02	21.42 ± 0.02	...	22.08 ± 0.04
22.26 ± 0.04	21.91 ± 0.03	22.44 ± 0.04	21.18 ± 0.01	21.73 ± 0.03	21.73 ± 0.02	21.36 ± 0.02	23.64 ± 0.10	22.14 ± 0.04
22.05 ± 0.04	21.96 ± 0.03	22.05 ± 0.03	21.01 ± 0.01	21.51 ± 0.02	21.46 ± 0.02	22.03 ± 0.04	23.80 ± 0.16	21.88 ± 0.04
22.05 ± 0.04	22.05 ± 0.04	22.07 ± 0.04	21.08 ± 0.02	21.55 ± 0.02	21.56 ± 0.03	22.12 ± 0.05	...	21.81 ± 0.04
21.99 ± 0.02	21.90 ± 0.02	21.92 ± 0.02	20.92 ± 0.01	21.38 ± 0.01	21.42 ± 0.01	22.02 ± 0.02	...	22.00 ± 0.02
21.97 ± 0.04	21.99 ± 0.03	22.05 ± 0.03	20.99 ± 0.01	21.34 ± 0.01	21.40 ± 0.02	21.84 ± 0.03	23.94 ± 0.13	22.05 ± 0.04
22.03 ± 0.02	22.08 ± 0.03	22.12 ± 0.02	21.02 ± 0.01	21.38 ± 0.01	21.44 ± 0.02	21.85 ± 0.03	23.90 ± 0.12	21.98 ± 0.04
22.01 ± 0.02	22.05 ± 0.03	22.08 ± 0.02	20.97 ± 0.01	21.34 ± 0.01	21.38 ± 0.01	21.85 ± 0.02	23.81 ± 0.10	22.05 ± 0.02
21.95 ± 0.02	22.06 ± 0.03	22.05 ± 0.03	21.01 ± 0.01	21.36 ± 0.02	21.39 ± 0.02	21.81 ± 0.03	23.89 ± 0.13	22.07 ± 0.04
22.01 ± 0.02	22.07 ± 0.02	22.10 ± 0.02	20.99 ± 0.01	21.36 ± 0.01	21.38 ± 0.01	21.78 ± 0.02	23.81 ± 0.06	22.02 ± 0.02
22.05 ± 0.04	22.18 ± 0.04	22.10 ± 0.04	20.95 ± 0.01	21.36 ± 0.02	21.35 ± 0.02	21.72 ± 0.03	23.59 ± 0.11	21.98 ± 0.04
22.00 ± 0.04	22.15 ± 0.04	22.05 ± 0.03	20.92 ± 0.01	21.33 ± 0.02	21.33 ± 0.02	21.69 ± 0.03	23.81 ± 0.13	22.02 ± 0.04
22.06 ± 0.02	22.21 ± 0.02	22.09 ± 0.02	20.98 ± 0.01	21.39 ± 0.01	21.35 ± 0.01	21.63 ± 0.01	23.69 ± 0.08	21.93 ± 0.02
22.19 ± 0.04	22.17 ± 0.04	21.94 ± 0.02	20.89 ± 0.01	21.55 ± 0.02	21.38 ± 0.02	21.41 ± 0.03	23.17 ± 0.08	21.91 ± 0.04
22.15 ± 0.04	22.09 ± 0.05	21.88 ± 0.03	20.83 ± 0.02	21.51 ± 0.03	21.28 ± 0.02	21.41 ± 0.03	23.49 ± 0.16	22.07 ± 0.04
22.16 ± 0.04	21.89 ± 0.03	21.98 ± 0.03	20.88 ± 0.01	21.67 ± 0.02	21.46 ± 0.02	21.23 ± 0.02	22.89 ± 0.07	21.95 ± 0.04
22.19 ± 0.02	21.85 ± 0.02	21.95 ± 0.02	20.78 ± 0.01	21.60 ± 0.01	21.45 ± 0.01	21.13 ± 0.01	22.86 ± 0.04	22.30 ± 0.04
22.22 ± 0.02	21.88 ± 0.02	21.98 ± 0.02	20.81 ± 0.01	21.60 ± 0.01	21.46 ± 0.01	21.08 ± 0.01	22.76 ± 0.03	22.26 ± 0.02
22.15 ± 0.04	21.92 ± 0.04	22.03 ± 0.04	20.89 ± 0.01	21.65 ± 0.03	21.61 ± 0.03	21.09 ± 0.02	22.81 ± 0.09	22.14 ± 0.05
21.95 ± 0.02	22.03 ± 0.03	22.31 ± 0.02	20.88 ± 0.01	21.56 ± 0.01	21.69 ± 0.01	21.02 ± 0.01	23.13 ± 0.05	21.99 ± 0.02
21.98 ± 0.02	21.98 ± 0.02	22.36 ± 0.03	20.90 ± 0.01	21.55 ± 0.02	21.66 ± 0.02	20.92 ± 0.01	23.33 ± 0.07	21.91 ± 0.02
22.12 ± 0.02	22.07 ± 0.02	22.42 ± 0.03	20.93 ± 0.01	21.55 ± 0.01	21.66 ± 0.01	20.80 ± 0.01	23.39 ± 0.06	21.78 ± 0.01
				g				
23.34 ± 0.16	22.73 ± 0.09	...	22.22 ± 0.05	22.45 ± 0.07	22.55 ± 0.07	21.89 ± 0.08	...	23.10 ± 0.12
...	...	...	22.06 ± 0.06	...	...	...	...	23.27 ± 0.17
23.08 ± 0.06	23.02 ± 0.08	23.56 ± 0.11	21.94 ± 0.05	22.19 ± 0.05	23.03 ± 0.11	22.55 ± 0.05	...	23.26 ± 0.09
23.16 ± 0.05	...	23.35 ± 0.05	22.40 ± 0.05	22.71 ± 0.05	22.98 ± 0.05	...	24.30 ± 0.14	23.45 ± 0.08
				i				
21.05 ± 0.05	21.52 ± 0.05	...	20.31 ± 0.05	20.91 ± 0.05	20.65 ± 0.05	20.09 ± 0.05	22.10 ± 0.10	21.30 ± 0.05
21.26 ± 0.05	21.41 ± 0.05	21.25 ± 0.05	20.47 ± 0.05	20.72 ± 0.05	20.94 ± 0.05	20.43 ± 0.05	21.85 ± 0.09	21.03 ± 0.05

TABLE 4—*Continued*

V18	V19	V20	V21	V22	V23	V24	V25	V26	V27
				r					
23.72 ± 0.14	20.81 ± 0.01	...	22.03 ± 0.03	21.84 ± 0.05	21.60 ± 0.03	20.80 ± 0.02	21.63 ± 0.04	21.67 ± 0.03	21.51 ± 0.03
...	...	...	...	...	...	...	...	...	...
...	...	...	22.10 ± 0.05	...	...	...	...	...	...
...	20.62 ± 0.05	...	22.01 ± 0.04	21.51 ± 0.04	21.99 ± 0.04	20.96 ± 0.02	21.60 ± 0.04	22.32 ± 0.06	22.12 ± 0.05
...	...	...	21.81 ± 0.03	21.94 ± 0.06	21.74 ± 0.04	20.69 ± 0.02	21.69 ± 0.04	21.88 ± 0.04	21.57 ± 0.03
...	20.53 ± 0.01	...	21.88 ± 0.03	20.97 ± 0.03	21.49 ± 0.02	21.12 ± 0.02	21.53 ± 0.02	21.90 ± 0.02	22.06 ± 0.04
23.78 ± 0.10	20.53 ± 0.01	21.23 ± 0.01	21.93 ± 0.02	21.02 ± 0.03	21.48 ± 0.02	21.09 ± 0.02	21.46 ± 0.02	21.88 ± 0.02	22.07 ± 0.03
23.65 ± 0.10	20.88 ± 0.02	21.10 ± 0.02	21.38 ± 0.02	20.88 ± 0.02	21.62 ± 0.02	20.74 ± 0.01	21.60 ± 0.02	21.62 ± 0.02	22.38 ± 0.04
23.56 ± 0.17	20.75 ± 0.02	21.05 ± 0.02	21.35 ± 0.02	20.74 ± 0.02	21.63 ± 0.03	20.64 ± 0.02	21.49 ± 0.04	21.50 ± 0.03	22.28 ± 0.06
23.61 ± 0.09	20.67 ± 0.01	21.05 ± 0.01	21.41 ± 0.01	20.84 ± 0.01	21.59 ± 0.02	20.65 ± 0.02	21.45 ± 0.01	21.28 ± 0.22	22.21 ± 0.04
23.51 ± 0.12	20.66 ± 0.01	21.03 ± 0.01	21.38 ± 0.02	20.73 ± 0.02	21.64 ± 0.03	20.66 ± 0.02	21.55 ± 0.04	21.52 ± 0.03	22.24 ± 0.06
23.86 ± 0.14	20.66 ± 0.01	21.08 ± 0.01	21.38 ± 0.03	...	21.66 ± 0.03	20.69 ± 0.02	21.52 ± 0.02	...	22.24 ± 0.04
23.59 ± 0.13	20.63 ± 0.01	21.05 ± 0.02	21.38 ± 0.02	20.81 ± 0.02	21.69 ± 0.03	20.67 ± 0.02	21.50 ± 0.02	21.67 ± 0.03	22.27 ± 0.05
24.15 ± 0.16	20.55 ± 0.01	21.06 ± 0.02	21.38 ± 0.02	20.90 ± 0.02	21.67 ± 0.02	20.68 ± 0.01	21.46 ± 0.02	...	22.18 ± 0.04
24.02 ± 0.18	20.73 ± 0.02	21.19 ± 0.02	21.85 ± 0.03	21.53 ± 0.03	21.32 ± 0.02	21.32 ± 0.03	21.61 ± 0.04	21.67 ± 0.03	21.59 ± 0.03
24.27 ± 0.14	20.83 ± 0.01	21.21 ± 0.01	21.92 ± 0.02	21.53 ± 0.02	21.39 ± 0.02	21.44 ± 0.01	21.61 ± 0.02	21.68 ± 0.02	21.59 ± 0.02
24.14 ± 0.12	20.72 ± 0.01	21.18 ± 0.01	22.06 ± 0.02	21.61 ± 0.02	21.41 ± 0.01	21.48 ± 0.02	21.61 ± 0.02	21.64 ± 0.02	21.55 ± 0.03
24.41 ± 0.26	20.73 ± 0.01	21.23 ± 0.01	21.94 ± 0.03	21.61 ± 0.04	21.40 ± 0.02	21.47 ± 0.02	21.62 ± 0.02	21.66 ± 0.02	21.62 ± 0.03
24.01 ± 0.20	20.95 ± 0.01	21.38 ± 0.02	21.61 ± 0.03	20.99 ± 0.02	21.30 ± 0.02	20.71 ± 0.01	21.80 ± 0.04	21.89 ± 0.03	21.49 ± 0.03
24.01 ± 0.18	20.82 ± 0.02	21.44 ± 0.04	21.62 ± 0.02	20.94 ± 0.02	21.32 ± 0.02	20.68 ± 0.02	21.80 ± 0.04	21.93 ± 0.06	21.44 ± 0.02
23.83 ± 0.08	...	21.42 ± 0.02	21.55 ± 0.01	21.01 ± 0.01	21.30 ± 0.01	20.74 ± 0.01	21.82 ± 0.02	21.95 ± 0.02	...
23.99 ± 0.15	20.86 ± 0.02	21.30 ± 0.02	21.56 ± 0.02	20.98 ± 0.01	21.22 ± 0.02	20.67 ± 0.02	21.80 ± 0.04	22.13 ± 0.03	21.51 ± 0.02
24.11 ± 0.14	20.83 ± 0.02	21.27 ± 0.02	21.62 ± 0.02	21.05 ± 0.01	21.22 ± 0.01	20.66 ± 0.01	21.76 ± 0.02	22.10 ± 0.02	21.48 ± 0.02
24.02 ± 0.12	20.89 ± 0.02	21.26 ± 0.01	21.61 ± 0.01	21.01 ± 0.01	21.24 ± 0.01	20.67 ± 0.01	21.78 ± 0.02	22.15 ± 0.02	21.51 ± 0.02
24.13 ± 0.16	20.89 ± 0.02	21.26 ± 0.02	21.64 ± 0.02	21.05 ± 0.01	21.25 ± 0.01	20.67 ± 0.01	21.80 ± 0.02	22.20 ± 0.03	21.55 ± 0.02
24.01 ± 0.08	20.89 ± 0.01	21.24 ± 0.01	21.61 ± 0.01	21.05 ± 0.01	21.23 ± 0.01	20.63 ± 0.01	21.78 ± 0.02	22.18 ± 0.03	21.49 ± 0.02
...	20.91 ± 0.02	21.17 ± 0.02	21.72 ± 0.03	21.09 ± 0.03	21.26 ± 0.02	20.65 ± 0.02	21.83 ± 0.04	22.10 ± 0.04	21.52 ± 0.02
23.83 ± 0.14	20.96 ± 0.01	21.14 ± 0.01	21.66 ± 0.02	21.06 ± 0.02	21.28 ± 0.01	20.67 ± 0.02	21.84 ± 0.04	22.16 ± 0.03	21.58 ± 0.02
23.90 ± 0.07	20.90 ± 0.01	21.16 ± 0.01	21.71 ± 0.01	21.10 ± 0.01	21.31 ± 0.01	20.62 ± 0.01	21.81 ± 0.02	22.15 ± 0.02	21.55 ± 0.02
23.69 ± 0.12	20.93 ± 0.02	21.03 ± 0.01	21.88 ± 0.02	21.27 ± 0.03	21.38 ± 0.02	20.72 ± 0.02	21.84 ± 0.02	22.01 ± 0.03	21.51 ± 0.02
23.70 ± 0.17	20.94 ± 0.02	21.01 ± 0.02	21.88 ± 0.03	21.31 ± 0.04	21.38 ± 0.02	20.71 ± 0.02	21.92 ± 0.04	22.00 ± 0.04	21.53 ± 0.03
...	20.96 ± 0.02	20.95 ± 0.01	22.07 ± 0.03	21.34 ± 0.02	21.50 ± 0.02	20.75 ± 0.02	21.95 ± 0.04	21.66 ± 0.02	21.40 ± 0.02
23.39 ± 0.09	...	20.98 ± 0.01	22.09 ± 0.02	21.36 ± 0.02	21.52 ± 0.02	20.82 ± 0.01	22.02 ± 0.04	21.53 ± 0.01	21.39 ± 0.02
23.60 ± 0.05	21.08 ± 0.01	21.00 ± 0.01	22.05 ± 0.02	21.42 ± 0.01	21.53 ± 0.01	20.80 ± 0.01	21.97 ± 0.02	21.48 ± 0.01	21.33 ± 0.02
23.58 ± 0.18	20.92 ± 0.02	21.00 ± 0.02	22.15 ± 0.05	21.42 ± 0.04	21.55 ± 0.03	20.80 ± 0.02	21.99 ± 0.04	21.39 ± 0.02	21.23 ± 0.02
23.88 ± 0.10	21.19 ± 0.01	21.55 ± 0.01	22.05 ± 0.02	21.38 ± 0.02	22.08 ± 0.02	21.28 ± 0.01	21.66 ± 0.02	21.63 ± 0.02	21.80 ± 0.03
23.89 ± 0.11	21.09 ± 0.01	21.48 ± 0.01	22.13 ± 0.02	21.45 ± 0.02	22.05 ± 0.03	21.47 ± 0.03	21.42 ± 0.01	21.80 ± 0.02	21.94 ± 0.03
23.72 ± 0.07	21.01 ± 0.01	21.44 ± 0.01	22.08 ± 0.02	21.60 ± 0.01	21.83 ± 0.02	21.47 ± 0.02	21.38 ± 0.01	21.80 ± 0.01	21.89 ± 0.02
				g					
...	21.55 ± 0.05	...	23.20 ± 0.12	...	23.43 ± 0.15	...	...	23.85 ± 0.24	...
...	...	...	22.97 ± 0.12	...	23.42 ± 0.22	...	...	...	...
...	...	...	23.22 ± 0.07	...	23.14 ± 0.09	...	22.98 ± 0.11	23.09 ± 0.07	...
24.98 ± 0.23	22.46 ± 0.05	22.27 ± 0.05	22.95 ± 0.05	22.64 ± 0.05	22.68 ± 0.05	22.17 ± 0.05	23.38 ± 0.09	23.43 ± 0.07	23.05 ± 0.05
				i					
...	19.95 ± 0.05	...	21.05 ± 0.05	20.15 ± 0.05	20.77 ± 0.05	20.42 ± 0.05	20.97 ± 0.05	20.84 ± 0.05	20.80 ± 0.05
23.05 ± 0.12	20.09 ± 0.05	20.38 ± 0.05	20.72 ± 0.05	20.17 ± 0.05	20.97 ± 0.05	20.22 ± 0.05	20.94 ± 0.05	20.75 ± 0.05	21.05 ± 0.05

TABLE 4—*Continued*

V28	V29	V30	V31	V32	V33	V34	V35	V36	V37
					r				
21.80 ± 0.04	22.28 ± 0.06	21.55 ± 0.03	22.46 ± 0.06	21.80 ± 0.04	21.16 ± 0.05	21.11 ± 0.02	21.67 ± 0.03	21.23 ± 0.03	...
...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...
22.67 ± 0.11	22.49 ± 0.07	21.82 ± 0.03	22.71 ± 0.07	22.22 ± 0.05	21.12 ± 0.02	21.52 ± 0.03	20.98 ± 0.01	21.76 ± 0.04	23.22 ± 0.12
21.52 ± 0.03	22.15 ± 0.05	21.95 ± 0.04	22.86 ± 0.10	22.12 ± 0.05	21.50 ± 0.04	21.45 ± 0.03	21.14 ± 0.02	21.49 ± 0.03	22.18 ± 0.06
21.38 ± 0.03	22.30 ± 0.04	22.27 ± 0.04	22.76 ± 0.05	22.27 ± 0.04	21.98 ± 0.02	...	...	...	...
21.34 ± 0.02	22.35 ± 0.04	22.40 ± 0.05	22.74 ± 0.06	22.19 ± 0.03	21.92 ± 0.04	21.86 ± 0.03	21.44 ± 0.02	21.80 ± 0.03	21.47 ± 0.02
21.46 ± 0.02	22.74 ± 0.04	21.56 ± 0.02	22.90 ± 0.05	...	...	21.94 ± 0.06	...	...	...
21.35 ± 0.03	22.66 ± 0.08	21.49 ± 0.03	22.60 ± 0.07	22.10 ± 0.05	21.84 ± 0.04	21.28 ± 0.02	21.42 ± 0.02	21.83 ± 0.03	...
21.27 ± 0.02	22.71 ± 0.04	21.46 ± 0.01	22.64 ± 0.03	21.91 ± 0.02	21.80 ± 0.02	21.27 ± 0.01	21.34 ± 0.01	21.75 ± 0.02	21.36 ± 0.02
21.34 ± 0.03	22.70 ± 0.06	21.46 ± 0.03	22.59 ± 0.05	21.98 ± 0.04	21.76 ± 0.04	21.33 ± 0.02	21.34 ± 0.02	21.82 ± 0.03	21.39 ± 0.02
21.50 ± 0.03	22.75 ± 0.06	21.56 ± 0.03	22.67 ± 0.05	21.95 ± 0.04	21.56 ± 0.04	21.33 ± 0.03	21.35 ± 0.02	...	21.49 ± 0.02
21.51 ± 0.03	22.78 ± 0.07	21.58 ± 0.03	22.62 ± 0.06	22.03 ± 0.04	21.55 ± 0.02	21.38 ± 0.02	21.35 ± 0.02	21.85 ± 0.03	21.48 ± 0.03
21.57 ± 0.03	22.88 ± 0.06	21.61 ± 0.02	22.73 ± 0.05	21.95 ± 0.03	21.41 ± 0.02	21.32 ± 0.02	21.30 ± 0.02	21.73 ± 0.04	...
21.91 ± 0.04	22.69 ± 0.06	21.59 ± 0.03	22.84 ± 0.06	21.81 ± 0.03	21.18 ± 0.02	21.15 ± 0.02	21.50 ± 0.02	21.40 ± 0.02	22.45 ± 0.05
21.41 ± 0.02	22.85 ± 0.04	21.51 ± 0.02	22.76 ± 0.04	21.83 ± 0.02	21.27 ± 0.01	21.21 ± 0.01	21.51 ± 0.02	21.41 ± 0.02	...
21.31 ± 0.02	22.77 ± 0.04	21.49 ± 0.02	22.67 ± 0.04	21.78 ± 0.02	21.17 ± 0.01	21.11 ± 0.01	21.38 ± 0.01	21.28 ± 0.01	22.09 ± 0.03
21.38 ± 0.02	22.88 ± 0.07	21.48 ± 0.02	22.70 ± 0.05	21.86 ± 0.03	21.30 ± 0.01	21.23 ± 0.02	21.50 ± 0.02	21.42 ± 0.02	22.27 ± 0.04
22.17 ± 0.04	22.42 ± 0.04	21.99 ± 0.03	22.67 ± 0.05	22.08 ± 0.04	20.86 ± 0.01	21.65 ± 0.02	21.14 ± 0.02	21.55 ± 0.02	21.73 ± 0.03
22.17 ± 0.04	22.49 ± 0.05	21.95 ± 0.04	22.61 ± 0.06	22.02 ± 0.03	20.80 ± 0.01	21.59 ± 0.03	21.06 ± 0.01	21.55 ± 0.03	21.55 ± 0.02
22.22 ± 0.03	22.39 ± 0.03	21.97 ± 0.02	...	22.08 ± 0.02	...	21.76 ± 0.02	...	...	...
21.58 ± 0.02	22.55 ± 0.05	21.76 ± 0.02	22.55 ± 0.08	22.11 ± 0.03	20.89 ± 0.01	21.77 ± 0.02	21.18 ± 0.01	21.39 ± 0.02	22.03 ± 0.03
21.40 ± 0.02	22.47 ± 0.04	21.74 ± 0.02	22.49 ± 0.03	22.06 ± 0.03	20.83 ± 0.01	21.71 ± 0.02	21.14 ± 0.01	21.31 ± 0.02	21.95 ± 0.02
21.44 ± 0.02	22.50 ± 0.03	21.72 ± 0.02	22.45 ± 0.03	22.09 ± 0.02	20.90 ± 0.01	21.80 ± 0.02	21.20 ± 0.01	21.38 ± 0.01	22.12 ± 0.03
21.42 ± 0.02	22.53 ± 0.05	21.73 ± 0.02	22.36 ± 0.03	22.20 ± 0.03	20.90 ± 0.01	21.80 ± 0.02	21.18 ± 0.01	21.34 ± 0.02	22.10 ± 0.03
21.33 ± 0.01	22.47 ± 0.05	21.72 ± 0.02	22.43 ± 0.03	22.08 ± 0.02	20.88 ± 0.01	21.72 ± 0.01	21.15 ± 0.01	21.30 ± 0.01	22.07 ± 0.02
21.18 ± 0.02	22.73 ± 0.07	21.67 ± 0.03	22.38 ± 0.05	22.15 ± 0.04	20.93 ± 0.01	21.68 ± 0.03	21.19 ± 0.02	21.23 ± 0.02	22.32 ± 0.06
21.23 ± 0.02	22.67 ± 0.05	21.69 ± 0.03	22.30 ± 0.04	22.10 ± 0.03	20.93 ± 0.01	21.72 ± 0.02	21.22 ± 0.02	21.24 ± 0.02	22.28 ± 0.04
21.16 ± 0.01	22.65 ± 0.03	21.68 ± 0.02	22.27 ± 0.02	22.05 ± 0.02	20.93 ± 0.01	21.66 ± 0.01	21.22 ± 0.01	21.24 ± 0.01	...
21.28 ± 0.02	22.21 ± 0.04	21.72 ± 0.02	22.24 ± 0.03	21.88 ± 0.02	20.92 ± 0.01	21.34 ± 0.02	21.32 ± 0.02	21.28 ± 0.02	22.72 ± 0.05
21.46 ± 0.03	22.17 ± 0.05	21.67 ± 0.03	22.27 ± 0.05	21.86 ± 0.04	21.05 ± 0.02	21.36 ± 0.02	21.40 ± 0.02	21.33 ± 0.03	22.85 ± 0.09
21.75 ± 0.03	22.12 ± 0.07	21.68 ± 0.03	22.49 ± 0.05	21.77 ± 0.02	21.07 ± 0.01	21.17 ± 0.02	21.39 ± 0.02	21.47 ± 0.02	22.96 ± 0.08
22.23 ± 0.03	22.30 ± 0.03	21.66 ± 0.02	22.70 ± 0.05	21.85 ± 0.02	21.31 ± 0.01	21.25 ± 0.01	21.55 ± 0.02	21.62 ± 0.02	...
22.24 ± 0.03	22.27 ± 0.02	21.65 ± 0.01	22.59 ± 0.03	21.78 ± 0.01	21.25 ± 0.01	21.19 ± 0.01	21.48 ± 0.01	21.53 ± 0.01	...
22.30 ± 0.06	22.28 ± 0.05	21.65 ± 0.03	22.58 ± 0.06	21.85 ± 0.04	21.24 ± 0.02	21.17 ± 0.02	21.43 ± 0.03	21.48 ± 0.03	...
21.50 ± 0.02	22.41 ± 0.03	22.05 ± 0.02	22.67 ± 0.04	22.27 ± 0.03	22.14 ± 0.04	21.81 ± 0.03	21.51 ± 0.02	21.31 ± 0.01	...
21.92 ± 0.03	22.70 ± 0.05	22.20 ± 0.03	22.75 ± 0.05	22.36 ± 0.03	22.20 ± 0.04	21.88 ± 0.02	21.46 ± 0.02	21.31 ± 0.01	...
22.08 ± 0.02	22.58 ± 0.03	22.18 ± 0.02	22.63 ± 0.03	22.40 ± 0.02	22.22 ± 0.02	21.78 ± 0.01	21.42 ± 0.01	21.21 ± 0.01	21.77 ± 0.02
					g				
...	23.57 ± 0.17	22.66 ± 0.09	...	23.35 ± 0.15	22.38 ± 0.06	22.88 ± 0.11	22.16 ± 0.05	23.55 ± 0.22	...
...	...	...	...	...	22.91 ± 0.12	...	22.39 ± 0.07	23.01 ± 0.17	...
...	23.46 ± 0.10	22.80 ± 0.08	...	23.20 ± 0.09	22.58 ± 0.05	22.91 ± 0.10	22.38 ± 0.05	22.70 ± 0.08	23.52 ± 0.09
23.01 ± 0.05	24.01 ± 0.12	...	24.09 ± 0.13	23.51 ± 0.08	22.38 ± 0.05	23.28 ± 0.05	...	23.22 ± 0.06	23.55 ± 0.07
					i				
21.35 ± 0.05	21.65 ± 0.05	21.61 ± 0.05	21.86 ± 0.06	21.22 ± 0.06	20.83 ± 0.05	20.80 ± 0.05	20.50 ± 0.05	20.76 ± 0.05	...
21.34 ± 0.05	21.97 ± 0.05	21.05 ± 0.05	21.83 ± 0.07	21.05 ± 0.05	20.73 ± 0.05	20.62 ± 0.05	20.49 ± 0.05	20.75 ± 0.05	...

TABLE 5

Cepheid Variables

Object (1)	Period (days) (2)	$\langle g \rangle$[a] (3)	$\sigma_{\langle g \rangle}$ (4)	$\langle r \rangle$[b] (5)	$\sigma_{\langle r \rangle}$ (6)	$\langle i \rangle$[c] (7)	$\sigma_{\langle i \rangle}$ (8)	X (9)	Y (10)	R.A. (J2000) (11)	Decl. (J2000) (12)
C01.....	18.65	23.59	0.21	23.45	0.09	23.47	0.24	177.17	931.06	12 26 16.8	33 33 07
C02.....	12.99	24.24	0.26	23.84	0.11	23.44	0.23	477.19	143.28	12 26 12.9	33 30 33
C03.....	15.06	24.13	0.25	23.55	0.10	22.82	0.19	863.10	198.74	12 26 06.3	33 30 42
C04.....	71.26	21.60	0.08	21.35	0.02	21.23	0.16	870.34	152.16	12 26 06.3	33 30 33
C05.....	26.20	23.12	0.19	23.26	0.08	23.18	0.21	962.14	780.46	12 26 03.9	33 32 35
C06.....	25.80	23.33	0.19	23.16	0.08	22.91	0.18	1493.71	878.36	12 25 54.9	33 32 51
C07.....	27.99	22.70	0.30	22.81	0.06	22.86	0.22	1551.86	69.44	12 25 55.0	33 30 14
C08.....	12.09	24.62	0.29	24.11	0.17	...	...	1567.74	346.60	12 25 54.3	33 31 07
C09.....	57.80	22.90	0.05	22.32	0.04	22.24	0.16	1612.33	419.98	12 25 53.5	33 31 22
C10.....	52.90	22.73	0.13	22.28	0.04	22.30	0.17	1647.97	1365.20	12 25 51.6	33 34 25
C11.....	89.89	22.03	0.03	21.56	0.02	21.57	0.16	1810.24	1237.91	12 25 49.1	33 34 00

[a] Mean g as computed in eq. (4).
[b] Phase-weighted intensity average.
[c] Mean i from r–i at i phase.

TABLE 6

Other Variable Stars

Object (1)	Period (days) (2)	$\langle g \rangle$[a] (3)	$\langle r \rangle$[a] (4)	$\langle i \rangle$[b] (5)	Δr[c] (6)	X (7)	Y (8)	R.A. (J2000) (9)	Decl. (J2000) (10)
V01......	570.0[d]	22.97	21.75	20.83	0.89	144.09	1154.40	12 26 17.1	33 33 51
V02......	207.8	24.82	23.39	22.36	0.92	208.96	360.19	12 26 17.1	33 31 16
V03......	211.0[d]	22.48	21.37	20.54	1.32	216.63	202.95	12 26 17.1	33 30 46
V04......	181.1	24.67	23.25	22.15	0.71	336.22	1638.35	12 26 13.2	33 35 24
V05......	237.9[d]	22.17	20.91	20.13	0.67	515.87	322.67	12 26 12.0	33 31 07
V06......	818.9[d]	22.84	21.70	20.94	1.13	651.49	536.32	12 26 09.4	33 31 48
V07......	680.1	23.40	22.22	21.53	0.49	731.36	339.72	12 26 08.3	33 31 10
V08......	505.3	23.03	21.60	20.90	0.48	953.77	282.99	12 26 04.7	33 30 58
V09......	456.8	23.23	22.14	21.23	0.68	988.34	360.40	12 26 04.0	33 31 13
V10......	305.7	22.86	21.95	21.33	0.42	1028.76	417.52	12 26 03.3	33 31 24
V11......	821.8[d]	23.45	22.09	21.17	0.70	1053.51	382.39	12 26 02.9	33 31 17
V12......	1500.0[d]	22.16	20.94	20.30	0.63	1139.67	617.67	12 26 01.1	33 32 02
V13......	570.8	22.35	21.46	21.08	0.67	1179.05	503.46	12 26 00.6	33 31 40
V14......	581.7	22.76	21.64	20.79	0.80	1236.18	637.74	12 25 59.5	33 32 06
V15......	818.5	22.17	21.32	20.39	1.29	1278.18	287.68	12 25 59.3	33 30 57
V16......	766.8[d]	24.30	23.35	22.42	1.17	1309.85	168.09	12 25 58.9	33 30 34
V17......	504.2[d]	23.28	21.88	21.06	0.78	1348.81	10.17	12 25 58.5	33 30 03
V18......	264.6	24.98	23.88	22.95	0.81	1361.41	1518.60	12 25 56.2	33 34 56
V19......	1024.7[d]	21.91	20.79	20.19	0.65	1406.63	49.68	12 25 57.4	33 30 10
V20......	411.4[d]	22.27	21.23	20.52	0.60	1427.24	1590.98	12 25 55.0	33 35 10
V21......	211.6[d]	23.07	21.78	20.95	0.76	1432.33	721.64	12 25 56.1	33 32 21
V22......	207.6[d]	22.64	21.38	20.54	0.92	1447.10	436.87	12 25 56.2	33 31 26
V23......	426.0[d]	22.98	21.54	20.86	0.80	1459.51	452.04	12 25 56.0	33 31 28
V24......	609.2	22.17	20.91	20.34	0.82	1462.68	326.57	12 25 56.1	33 31 04
V25......	1014.7	23.16	21.66	21.11	0.59	1463.24	235.67	12 25 56.2	33 30 46
V26......	450.3[d]	23.28	21.74	20.80	0.78	1565.12	473.45	12 25 54.2	33 31 32
V27......	509.7[d]	23.04	21.68	20.48	0.93	1589.24	304.76	12 25 54.0	33 30 59
V28......	184.6	23.01	21.65	21.49	1.09	1599.61	261.24	12 25 53.9	33 30 51
V29......	333.2	23.67	22.37	21.68	0.64	1615.12	945.73	12 25 52.7	33 33 04
V30......	447.7	22.73	21.80	21.25	0.84	1621.51	750.46	12 25 52.9	33 32 26
V31......	537.9	24.09	22.62	21.74	0.57	1689.84	1262.81	12 25 51.1	33 34 05
V32......	443.7	23.34	22.04	21.11	0.57	1706.81	705.39	12 25 51.5	33 32 17
V33......	545.2[d]	22.57	21.37	20.50	1.41	1790.41	1396.61	12 25 49.2	33 34 31
V34......	246.2	22.92	21.49	20.63	0.77	1806.01	1176.02	12 25 49.2	33 33 48
V35......	555.7	22.27	21.34	20.48	0.51	1823.23	651.81	12 25 49.7	33 32 06
V36......	413.5	23.19	21.48	20.45	0.56	1831.45	931.76	12 25 49.1	33 33 00
V37......	604.8	23.53	21.91	...	1.75	2007.94	1209.67	12 25 45.8	33 33 53

[a] Phase-weighted intensity average.
[b] Mean i from r–i at i phase.
[c] $\Delta r = r$ peak-to-peak amplitude.
[d] Period uncertain.

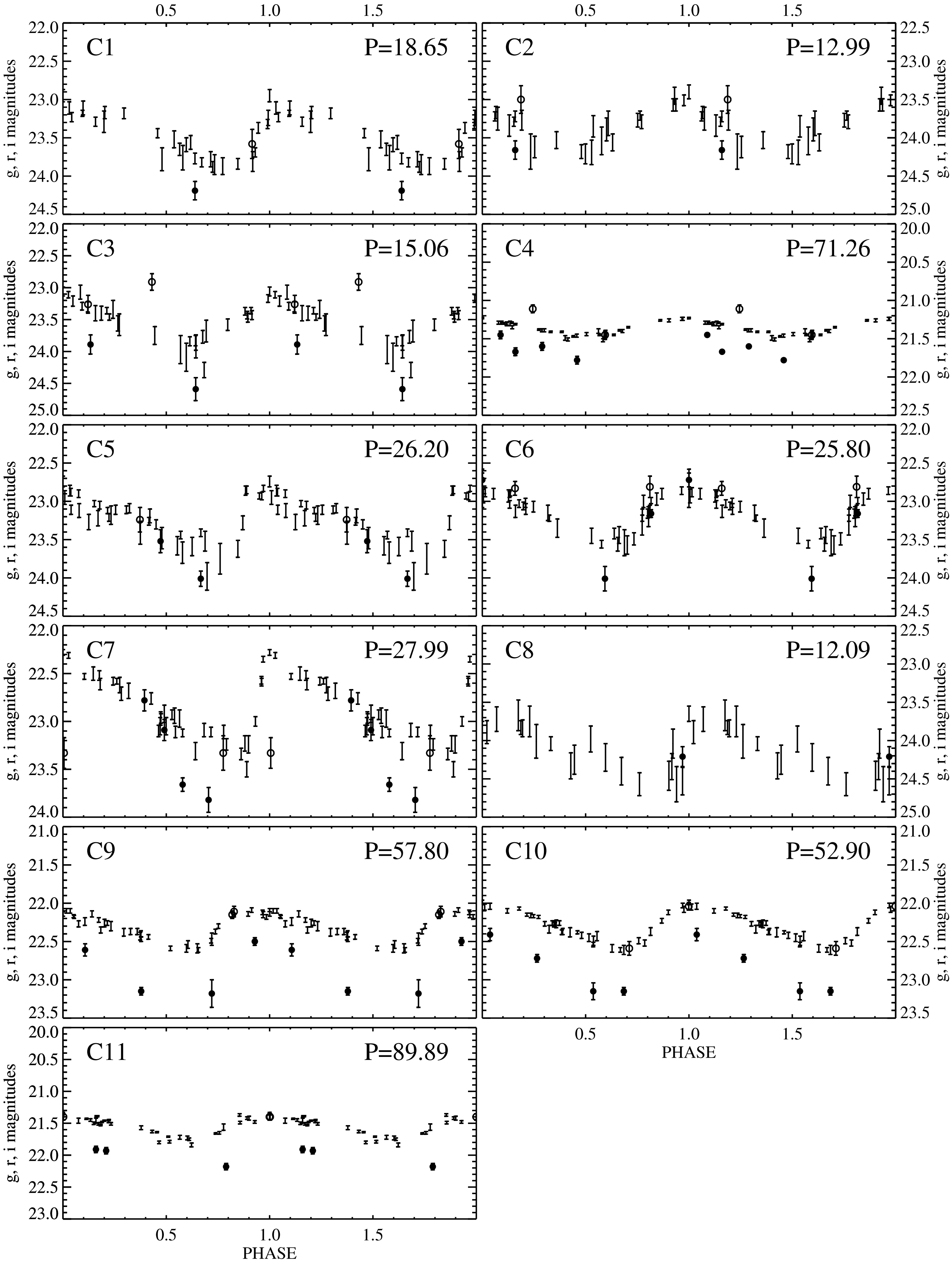

FIG. 2.—Light curves of the Cepheids in *r* (*small dots*), in *g* (*filled circles*), and in *i* (*open circles*). The identification numbers (C1–C11) and periods are displayed in each graph. The periods are given in days.

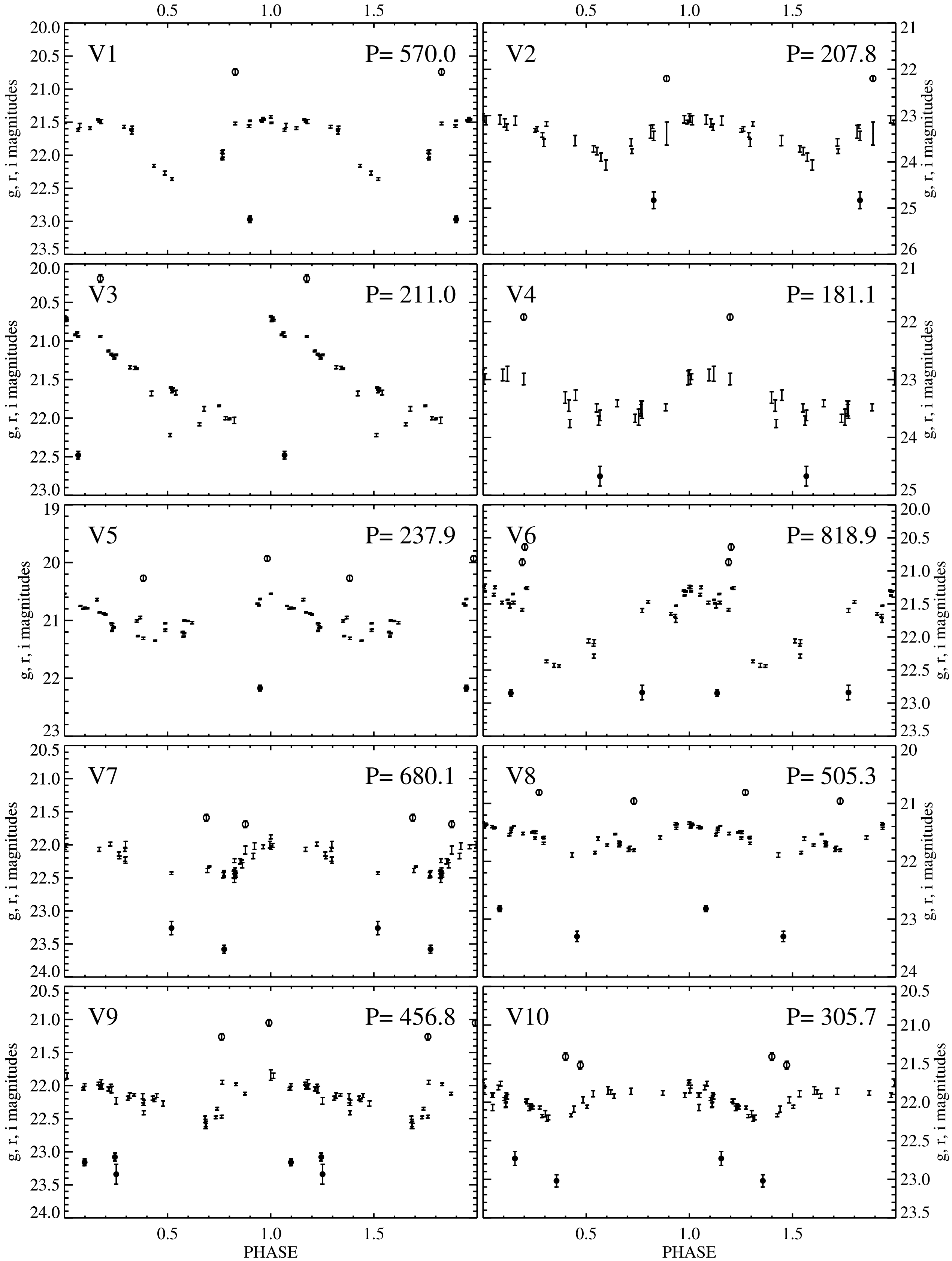

FIG. 3.—Light curves of the long-period variables in *r* (*small dots*), in *g* (*filled circles*), and in *i* (*open circles*). The identification numbers (V1–V37) and periods are displayed in each graph. The periods are given in days.

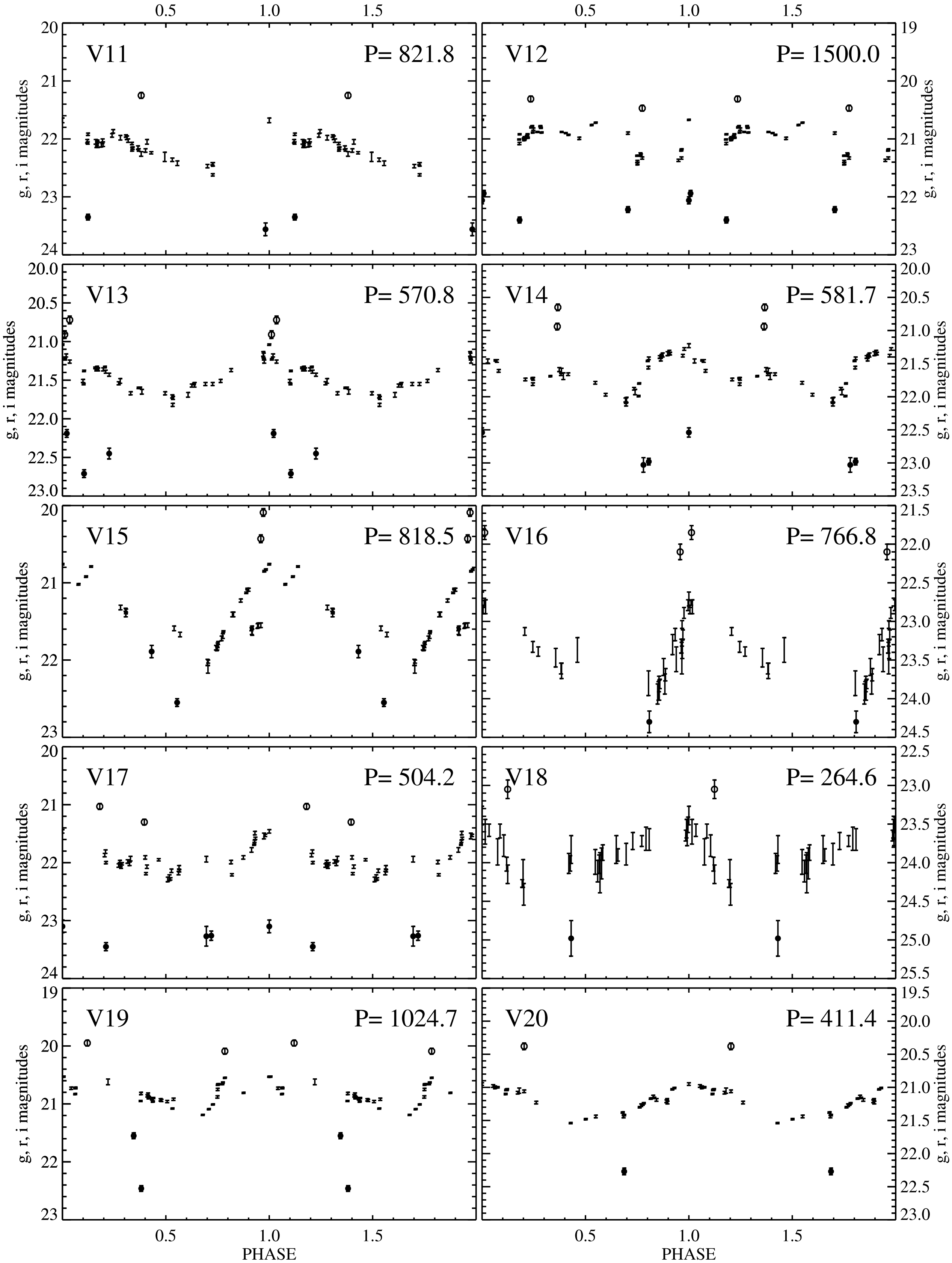

FIG. 3.—*Continued*

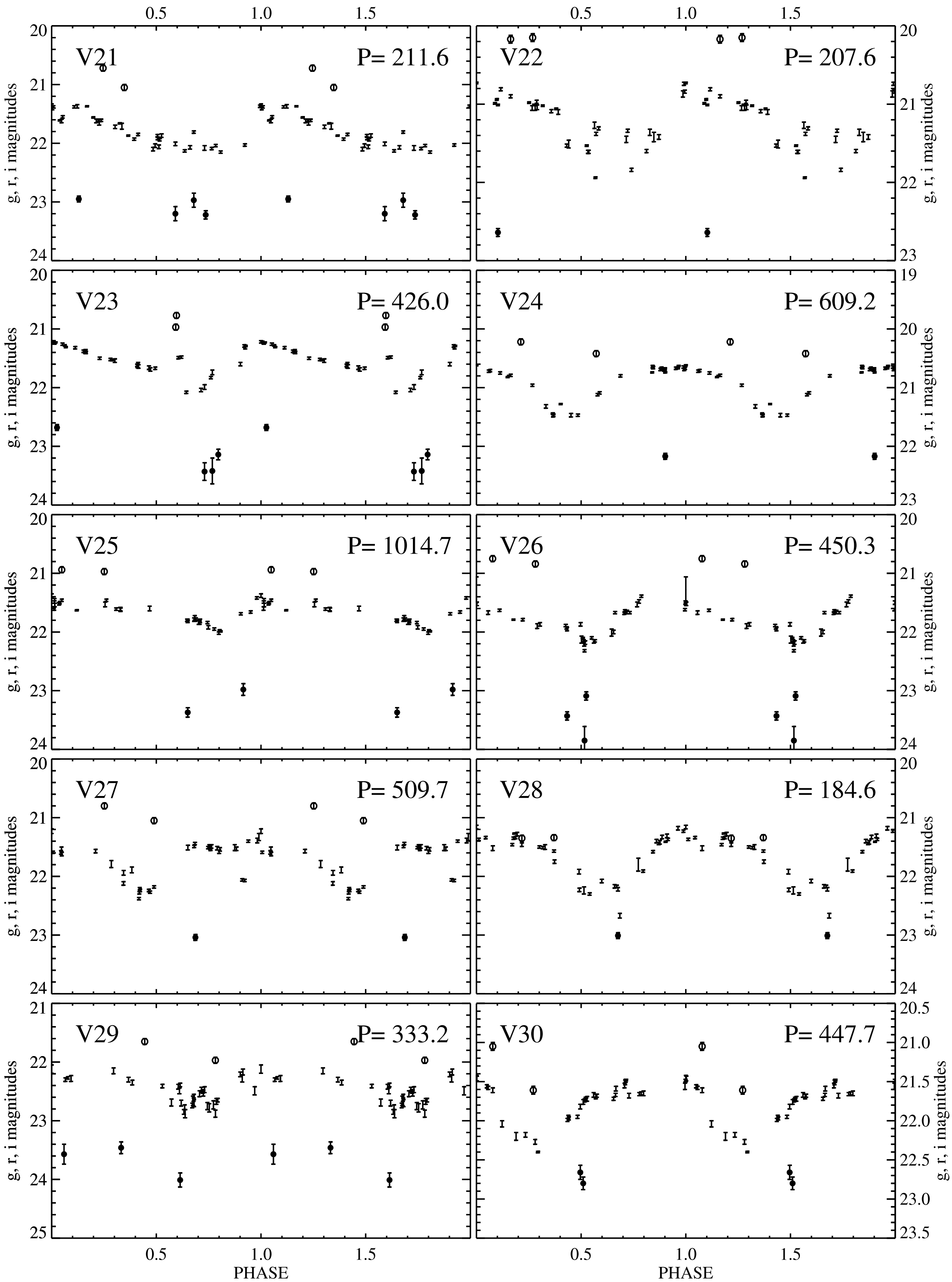

FIG. 3.—*Continued*

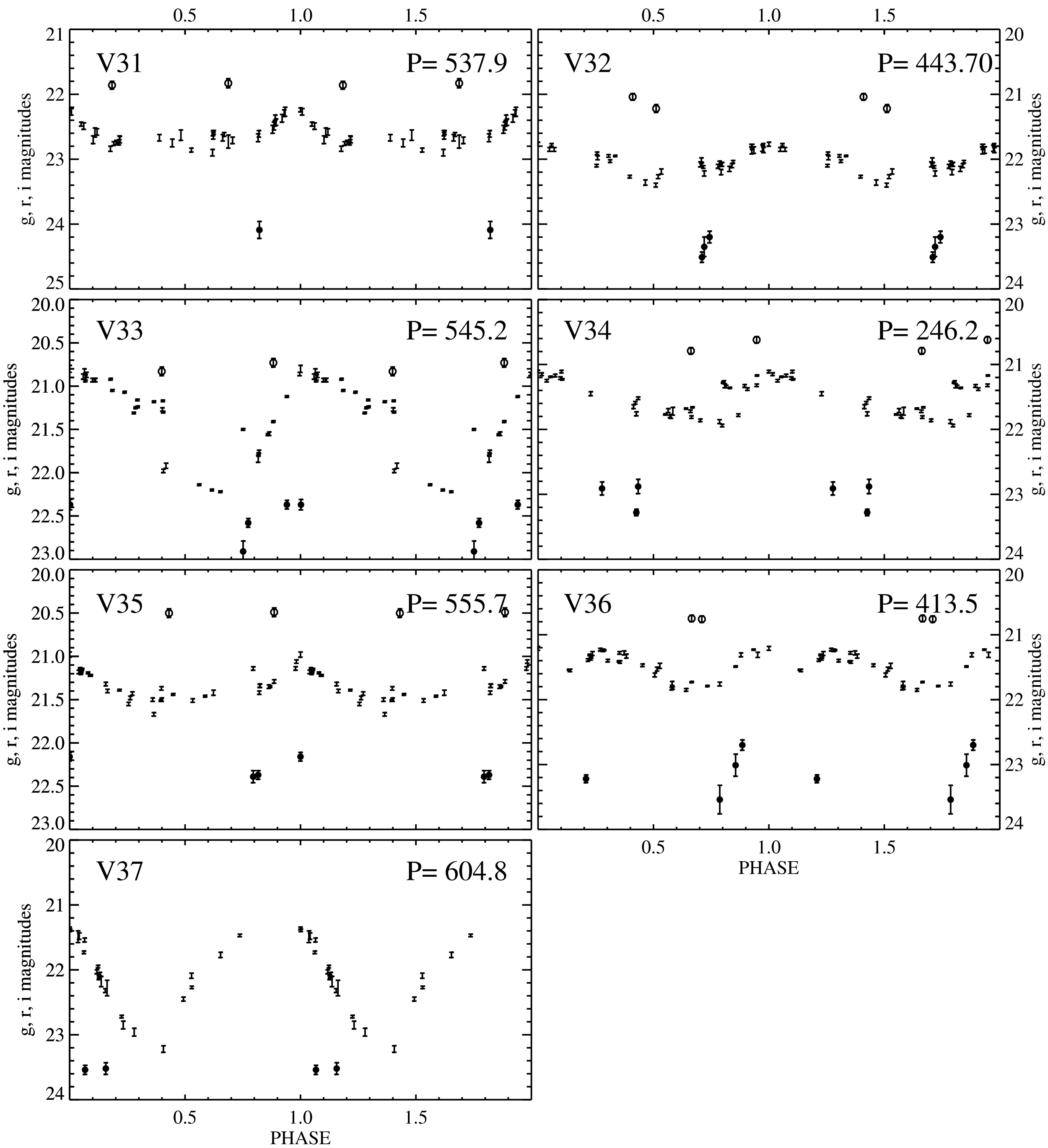

FIG. 3.—*Continued*

following transformations from the Johnson-Cousins-Landolt system to the Thuan-Gunn system:

$$g{-}V = -0.102 + 0.393(B{-}V), \quad (12)$$

$$r{-}R = 0.437 - 0.033(V{-}R), \quad (13)$$

$$i{-}I = 0.816 - 0.081(R{-}I). \quad (14)$$

Because these transformations were computed over a limited range of temperature, gravity, and metallicity, no second-order color terms are necessary. Naturally, this also means that these transformations are useful only for stars in this restricted range of parameters. Because the range in color of the Cepheids under consideration is quite limited, the linear approximation in equations (12)–(14) is sufficient.

Applying these synthetic transformations to the P-L relations of Madore & Freedman (1991) leads to the P-L relations in the Gunn system:

$$M_g = -2.63(\log P - 1) - 4.00, \quad (15)$$

$$M_r = -2.95(\log P - 1) - 4.09, \quad (16)$$

$$M_i = -3.07(\log P - 1) - 4.08. \quad (17)$$

Equations (16) and (17) (r and i bands) compare favorably with their counterparts (eqs. [10] and [11]) from Hoessel et al. (1994), based on the Kent (1985) transformations. However, in the g band (eq. [15] compared to eq. [9]), the difference is quite significant. The effect in terms of measured apparent distance moduli is

$$\delta\mu_g = 0.01(\log P - 1) - 0.08, \quad (18)$$

$$\delta\mu_r = 0.04(\log P - 1) + 0.05, \quad (19)$$

$$\delta\mu_i = 0.07(\log P - 1) + 0.02, \quad (20)$$

where the sign is the distance modulus from equations (15)–(17) minus that of Hoessel et al. (1994). For a Cepheid with a period of 25 days, the differences are −0.08 mag in g, +0.07 mag in r, and +0.04 mag in i. The synthetic transformations are superior to the older ones, since they are based on empirical observations of stars that are mostly dwarfs and whose SEDs differ systematically from supergiants. Our previous studies were based on ri photometry, and thus the color errors were small enough to be unnoticeable—but here, when the g band is used, the synthetic transformations are necessary, particularly since we will see in § 6 that the Kent transformations imply reddening values that are inconsistent with the observed colors of the bluest stars in NGC 4395.

The observed P-L relations are shown in Figure 4 for the g and r bands. Since the slope is fixed, the remaining free parameter is the offset that corresponds to the apparent distance moduli in g and r. The dashed lines indicate the intrinsic width of the P-L relation. The reported uncertainties are the standard errors, i.e., the rms scatter divided by the square root of the number of Cepheids.

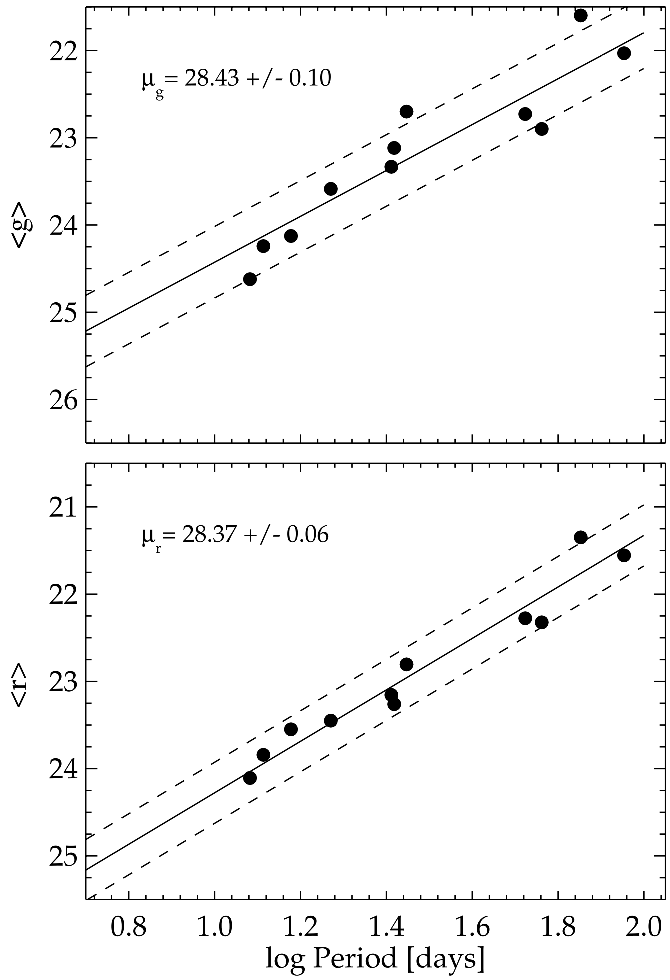

Fig. 4.—P-L relation of NGC 4395 in g (*top*) and r (*bottom*) for all 11 Cepheids. The solid lines represent the best fit with a slope of −2.63 in g and −2.95 in r from eqs. (15) and (16). The dashed lines account for an adopted intrinsic width of the instability strip of ±0.41 mag in g and ±0.35 mag in r.

6. EXTINCTION AND THE DISTANCE MODULUS

In order to derive the absorption-reddening relations

$$R_{g,r} = \frac{A_{g,r}}{E(g-r)}, \quad (21)$$

we have synthesized "g" and "r" magnitudes for Cepheid spectra as in § 5.2, but now including extinction using the equations of Cardelli, Clayton, & Mathis (1989). We adjusted the value of R_V to be that for which $A_V/E(B{-}V)$ is 3.1.

By comparing the results from different amounts of input absorption, we derived $A_g/E(g{-}r) = 3.44$. The true distance modulus is then given by

$$\mu_0 = \mu_g - A(g) = 3.44\mu_r - 2.44\mu_g \quad (22)$$

(cf. Tammann, Sandage, & Reindl 2003, eq. [40]).

We have applied the transformed P-L$_{g,r}$ relations in equations (15) and (16), which are based on the old P-L$_{B,V,R}$ relations of Madore & Freedman (1991), to the 11 Cepheids in NGC 4395. The resulting apparent distance moduli, μ_g and μ_r, are shown for each star in Table 7, as well as the individual true distance moduli from equation (22). The mean true distance modulus of NGC 4395 is then

$$\mu_0 = 28.22 \pm 0.21. \quad (23)$$

The apparent moduli μ_g and μ_r in Table 7 imply a mean reddening of the Cepheids of $E(g{-}r) = 0.059$.

If we use the P-L relation of Madore & Freedman (1991) with transformations by Kent (1985), the true distance modulus μ_0

TABLE 7
INDIVIDUAL DISTANCE MODULI AND REDDENING VALUES OF CEPHEIDS IN NGC 4395 USING g AND r MAGNITUDES

ID	P-L Relation with Transformation by Kent[a]				P-L Relation with Synthetic Transformation[b]			
	μ_g	μ_r	$E(g-r)$	μ_0	μ_g	μ_r	$E(g-r)$	μ_0
C01	28.375	28.278	0.097	28.040	28.298	28.339	−0.041	28.438
C02	28.620	28.213	0.407	27.219	28.541	28.267	0.273	27.600
C03	28.672	28.107	0.565	26.727	28.594	28.164	0.430	27.114
C04	27.913	27.871	0.042	27.769	27.841	27.955	−0.114	28.233
C05	28.293	28.518	−0.225	29.068	28.217	28.585	−0.368	29.483
C06	28.491	28.393	0.099	28.152	28.416	28.459	−0.044	28.566
C07	27.951	28.146	−0.195	28.621	27.876	28.214	−0.338	29.039
C08	28.917	28.387	0.530	27.093	28.838	28.440	0.398	27.470
C09	28.975	28.580	0.395	27.616	28.903	28.661	0.242	28.070
C10	28.703	28.421	0.282	27.733	28.631	28.500	0.130	28.182
C11	28.609	28.370	0.239	27.786	28.539	28.459	0.081	28.261
Mean	28.502 ± 0.104	28.299 ± 0.061	0.203 ± 0.081	27.802 ± 0.202	28.427 ± 0.104	28.367 ± 0.062	0.059 ± 0.081	28.223 ± 0.205

[a] P-L relation of Madore & Freedman (1991) with transformations by Kent (1985).
[b] P-L relation of Madore & Freedman (1991) with synthetic transformations.

would be 27.80, with a large reddening $E(g–r)$ of 0.20 (see Table 7). An examination of our color-magnitude diagram (CMD) (Fig. 6) indicates that the bluest stars are too blue to have a reddening of $E(g-r) = 0.20$, as the blue edge of the CMD is roughly 0.1 mag redward of the theoretical limit of $g-r = -0.8$, which therefore means that $E(g–r)$ cannot be greater than 0.1. This convinces us that the P-L relations from the synthetic transformations are more accurate, and we adopt them here.

The true distance modulus to NGC 4395 and the reddening can also be obtained from the mean r and i magnitudes of the Cepheids. The individual mean magnitudes are listed in Table 5. As shown in Table 3, the number of observations in the i band is one or two, except C08, which has no i magnitude. The mean i magnitude is calculated from the $r–i$ color at the i phase. Using again the P-L relation of Madore & Freedman (1991), the transformations by Kent (1985) for the r band, and the transformations by Wade et al. (1979) for the i band, we obtain $\mu_i = 28.20 \pm 0.12$ and $E(r-i) = 0.09 \pm 0.08$. If we use the P-L relation of Madore & Freedman (1991) with the synthetic transformations, the mean μ_i is 28.26 ± 0.13 with a mean $E(r–i)$ reddening of 0.10 ± 0.09. The individual values are presented in Table 8. The true distance modulus is given by

$$\mu_0 = 3.72\mu_i - 2.72\mu_r \quad (24)$$

(Saha et al. 2002, eq. [10]), which leads to $\mu_0 = 27.96 \pm 0.34$ and 27.98 ± 0.34 for the Kent-Wade and the synthetic transformations, respectively. This agreement demonstrates that the Kent-Wade transformations for r and i used in previous papers are consistent with the new synthetic relations. The estimated error in the present case, from using r and i, is much larger than that from using g and r, because there are fewer i than g observations. Below, we exclusively use the distances obtained with the synthetic transformations. We calculate the assumed true distance to NGC 4395 with the weighted mean of the true distances obtained with g and r on one hand, and the distance obtained with r and i on the other hand. The true weighted distance modulus is then given by combining both sets of distances,

$$\mu_0 = 28.15 \pm 0.18, \quad (25)$$

using the Madore & Freedman (1991) P-L relations.

TABLE 8
INDIVIDUAL DISTANCE MODULI AND REDDENING VALUES OF CEPHEIDS IN NGC 4395 USING r AND i MAGNITUDES

ID	P-L Relation with Transformation by Kent[a]				P-L Relation with Synthetic Transformation[b]			
	μ_r	μ_i	$E(r-i)$	μ_0	μ_r	μ_i	$E(r-i)$	μ_0
C01	28.278	28.342	−0.064	28.517	28.339	28.381	−0.042	28.497
C02	28.213	27.841	0.372	26.830	28.267	27.869	0.398	26.785
C03	28.107	27.414	0.693	25.529	28.164	27.446	0.718	25.494
C04	27.871	27.849	0.022	27.788	27.955	27.928	0.027	27.856
C05	28.518	28.495	0.023	28.431	28.585	28.544	0.041	28.433
C06	28.393	28.205	0.188	27.694	28.459	28.254	0.206	27.694
C07	28.146	28.261	−0.115	28.574	28.214	28.312	−0.099	28.581
C08	28.387	...	...	...	28.440	...	...	...
C09	28.580	28.586	−0.006	28.601	28.661	28.659	0.002	28.655
C10	28.421	28.530	−0.109	28.827	28.500	28.601	−0.101	28.875
C11	28.370	28.491	−0.121	28.820	28.459	28.578	−0.119	28.903
Mean	28.299 ± 0.061	28.201 ± 0.121	0.088 ± 0.083	27.961 ± 0.335	28.367 ± 0.062	28.257 ± 0.125	0.103 ± 0.085	27.977 ± 0.344

[a] P-L relation of Madore & Freedman (1991) with transformations by Kent (1985).
[b] P-L relation of Madore & Freedman (1991) with synthetic transformations.

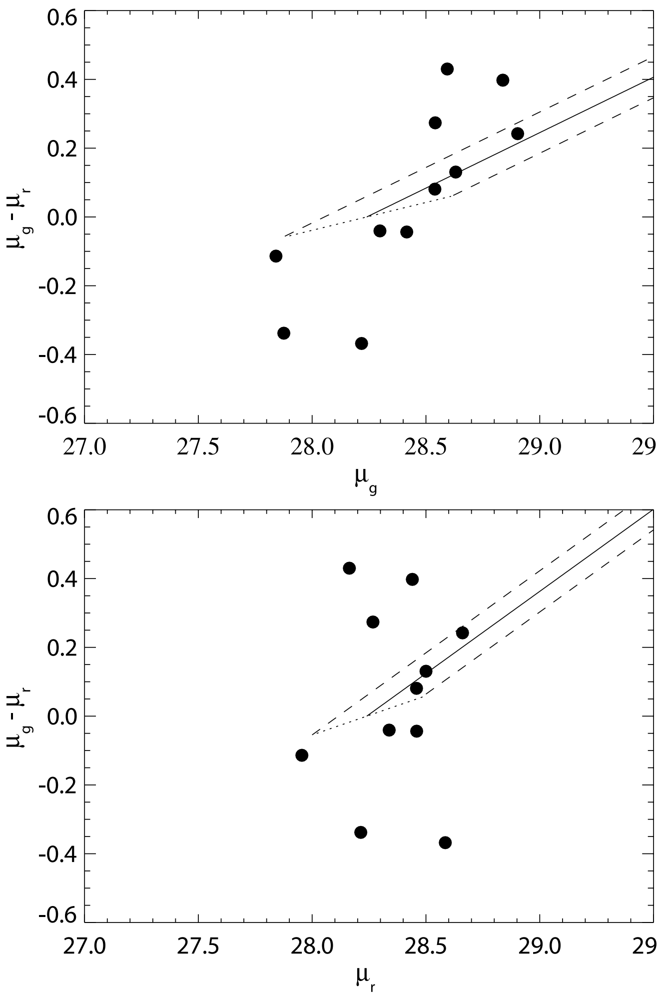

Fig. 5.—Individual apparent distance moduli μ_g (*top*) and μ_r (*bottom*) plotted against the difference in the apparent distance moduli in g and r for each Cepheid, which allows detection of differential absorption. The bold lines indicate the reddening vectors; the dashed lines indicate the width of the instability strip. With zero $E(g-r)$ reddening, the P-L relation is represented by a single point at $\mu_g = \mu_r = 28.22$. Because of the width of the instability strip, this point is elongated and is shown as a dotted line in both figures. It is sloped because of the change of color across the instability strip; for details, see Saha et al. (1996). The observed points in the top panel are spread along a slope of 1 rather than on the reddening line; in the bottom panel, the points seems to spread in the vertical direction. The distribution of the points illustrates that they do not lie on the reddening lines, which indicates that there is no measurable differential absorption at the level of the rms scatter in μ_r.

The Galactic reddening of $E(g-r) < 0.02$ (Burstein & Heiles 1984; Schlegel, Finkbeiner, & Davis 1998) cannot account for the full reddening of the Cepheids. They must suffer additional reddening in the parent galaxy. The diagnostic diagram in Figure 5 suggests that the intrinsic reddening is quite uniform.

A new fundamental problem with Cepheid distances arises from the fact that P-L relations in different galaxies are different (Tammann et al. 2002). The Galactic P-L relation is markedly steeper than that of the LMC; the latter is also nonlinear (Tammann et al. 2003). Tammann et al. showed that at least part of these differences are due to metallicity differences. If metallicity alone decides the form of the P-L relation, NGC 4395 with $12 + \log \mathrm{O/H} = 8.33 \pm 0.25$ (Roy et al. 1996) is similar to LMC and should be compared with the LMC P-L relation. The latter has been derived by Sandage

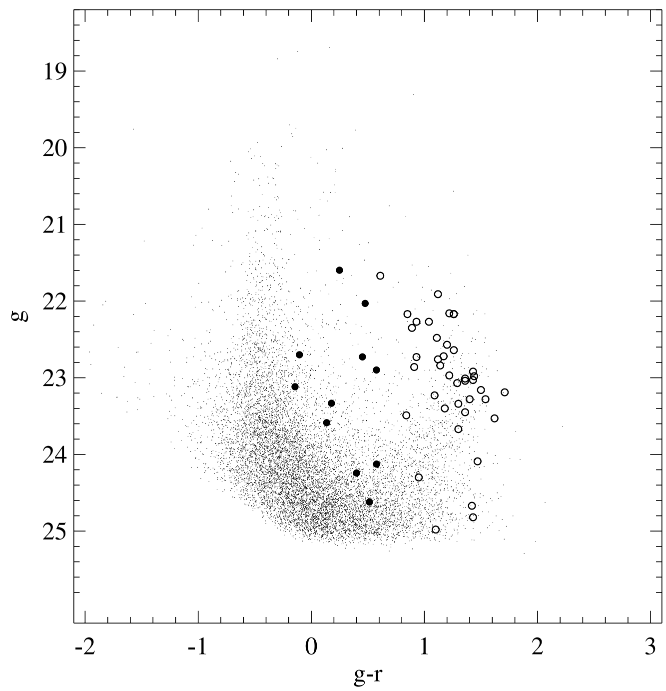

Fig. 6.—g vs. $g-r$ CMD. Cepheids are plotted as filled circles; other variable candidates are plotted as open circles.

et al. (2004) from the vast data of Udalski et al. (1999), which are augmented from external sources for longer period Cepheids. The new LMC P-L relations for Cepheids with periods more than 10 days in V and I are given by

$$M_V = -(2.609 \pm 0.099) \log P - (1.565 \pm 0.131), \qquad (26)$$

$$M_I = -(2.864 \pm 0.082) \log P - (2.010 \pm 0.108). \qquad (27)$$

Therefore, the true final dereddened distance modulus becomes

$$\mu_0 = 28.02 \pm 0.18. \qquad (28)$$

We prefer this value over $\mu_0 = 28.26 \pm 0.18$, which one would obtain from the P-L relation of the Galaxy, which is clearly more metal-rich than NGC 4395.

7. THE COLOR-MAGNITUDE DIAGRAM, EXTINCTION ESTIMATES, AND LPVS

7.1. *The Color-Magnitude Diagram and Extinction Estimates*

The color-magnitude diagrams (CMDs) for the template frames for g, r and r, i are shown in Figures 6 and 7. The large filled circles mark the position of the mean magnitudes of the Cepheids; open large circles indicate the position of the mean magnitudes of all other variable stars. The position of the Cepheids in the CMDs is consistent with expectation. The majority of other variables have red colors and periods longer than hundreds of days. They are listed in Table 6, and their light curves are presented in Figure 3.

As stated before, an examination of our CMD indicates that the bluest stars are too blue to have a reddening of $E(g-r) = 0.20$, as the blue edge of the CMD is roughly 0.1 mag redward of the theoretical limit of $g-r = -0.8$. Thus we believe the P-L relations from the synthetic transformations to be more accurate and adopt them in this paper.

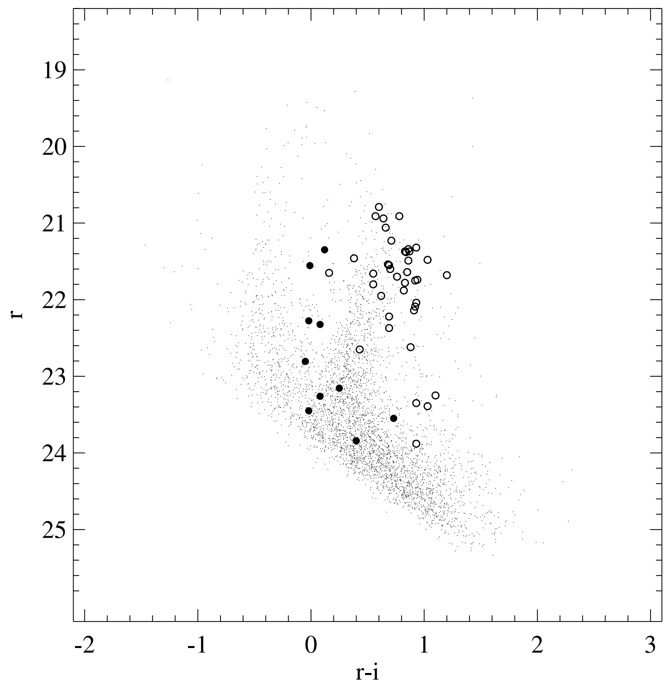

FIG. 7.—r vs. $r-i$ CMD. Cepheids are plotted as filled circles; other variable candidates are plotted as open circles.

7.2. *The Long-Period Variables*

Kholopov et al. (1985) defined three types of variable red giant stars. First, the Mira-type variables are long-period variables, with periods between 80 and 1000 days and amplitudes in the V band between 2.5 and 11 mag. Second, the semiregular variables have smaller amplitudes, from several hundreths to 2 mag in the V band, their period range is 20–2000 days or more, and their light curves show less regular behavior. The third group is the irregular variables, which have amplitudes of the order of 1 mag in the V band. Kholopov et al. also mention that many stars are misclassified as irregular variables because of incomplete studies.

The Kholopov et al. (1985) classification is based mainly on stars observed within our Galaxy. The selection effects for Galactic variable stars is very different from those for external galaxies. Within our Galaxy, fainter, relatively nearby variables abound, whereas very luminous variables, which are also short lived, are relatively rare, since they occur in the disk and are often obscured behind a lot of dust. On the other hand, in external galaxies these are much more easily seen, since they are among the brightest stars. In a near face-on case like in NGC 4395, extinction and obscuration are less significant. A better template for comparing the LPVs is from the M33 sample of Kinman, Mould, & Wood (1987). In their analysis, Kinman et al. (1987) identified red long-period variables as either core helium burning or upper asymptotic giant branch (AGB) stars. Core helium burning, or red supergiant (RSG) stars are at least 1 mag brighter than AGB stars (Wood, Bessell, & Fox 1983). The RSG long-period variables have smaller amplitudes than the AGB stars (Wood & Bessell 1985). In the r band, typical amplitudes for RSG variables are 1 mag or less, whereas for the AGB, LPV amplitudes as large as 3 mag are quite common. Furthermore, an AGB star has a degenerate core, and its luminosity is determined by its core mass, which in turn is subject to the Chandrasekhar limit. This implies an upper limit on the luminosity of an AGB star of $M_{\rm bol} \approx -7.0$, which is equivalent to $M_r = -5.20$ mag (Kinman et al. 1987).

To see if the above criteria can be used to distinguish between AGB and RSG LPVs, consider Figure 8, in which the mean r magnitudes of all possible long-period variables are plotted against their r amplitude. A clear break in the $\langle r \rangle$ magnitude distribution can be seen: there are four LPVs fainter than $\langle r \rangle = 23.2$, with the rest mostly brighter than $\langle r \rangle = 22$ mag. From the apparent distance modulus in r of 28.36 (see § 6), we conclude that an AGB star can be no brighter than $r = 23.16$. We therefore argue that the four LPVs in Figure 8 fainter than $r = 23.1$ are AGB stars, whereas the remainder are RSG LPVs. This argument is borne out by the location of these stars on the CMD obtained from our image data, as discussed in § 7.1.

However, the amplitudes of these two "classes" of LPVs do not follow expected behavior. The variables deduced to be AGB stars all have r amplitudes near 1.0 mag, which is smaller than normal. All but four of the putative RSG variables have reasonably small amplitudes, but the four with $\Delta r > 1.2$ mag clearly stand out in Figure 8. This is not entirely without precedent, since Hoessel, Saha, & Danielson (1998) found three very high amplitude red variables in the dwarf galaxy DDO 187, where these stars are among the brightest stars in that galaxy. Hoessel et al. (1998) discussed the possibility that these were not the "usual" RSG variables, but possibly the evolved products of Hubble-Sandage variables, with pulsations driven by Kelvin-Helmholtz instabilities as predicted by Heger et al. (1997). In the case of the four objects in question here, V3, V15, V33, and V37, the amplitudes, while larger than expected for RSG LPVs, are much smaller than the three stars mentioned in DDO 187. Heger et al. (1997) expect periods near 900 days for the red descendants of Hubble-Sandage variables, if they are indeed driven by Kelvin-Helmholtz instabilities as they predict, but this is not the case for the four stars at hand, which have periods from 210 to 820 days. We must stress that the *empirical* behavior of RSG LPVs is not known very well, and the moderately larger amplitudes for the four objects discussed here may not be so unusual after all. Only a systematic study of LPVs among young stellar populations with different metallicities can give us the answer.

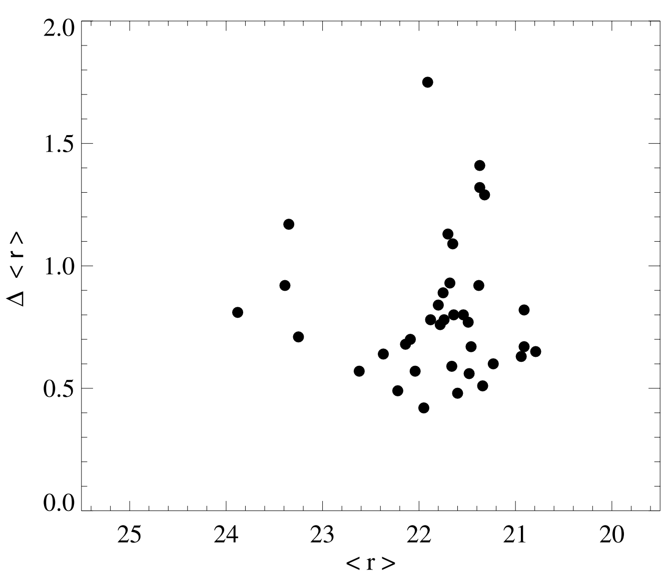

FIG. 8.—Mean r magnitudes for the long-period variables vs. r amplitude

8. SUMMARY

We have presented the results of a search for variable stars in NGC 4395 using the WIYN 3.5 m and the KPNO 2.1 m telescopes. Since the observations have been accumulated over a time span of 8 yr, we have been able to discover not only 11 Cepheids, but also 37 long-period variables (LPVs). A true distance modulus of 28.02 ± 0.18 and a mean reddening $E(g{-}r)$ of 0.06 and of $E(r{-}i)$ of 0.10 have been derived from the apparent distance moduli in *g*, *r*, and *i* based on the LMC P-L relation of Sandage et al. (2004). The dereddened distance modulus corresponds to a distance of 4.0 ± 0.3 Mpc. This adds to the growing list of galaxies well outside the Local Group for which Cepheid distances have been obtained from ground-based observations. NGC 4395 is of particular interest, since it is the nearest known Seyfert 1 galaxy.

Using an upper limit on the luminosity of an AGB star of $M_r \approx -5.2$ (Kinman et al. 1987) and a distance modulus of 28.02, we concluded that 4 out of the 37 red long-period variables are AGB stars, the rest probably being RSG LPVs.

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